

This policy has been largely adapted from the White Rose Calculation Policy with further material added. It is a working document and will be revised and amended as necessary.

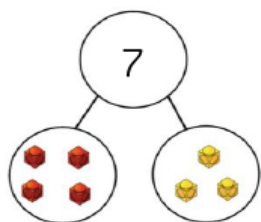


**GLENMERE PRIMARY SCHOOL
CALCULATION POLICY
YR 1-6
2023**

This policy begins with an overview of the different models and images that support the teaching of different Mathematical concepts and what we use at Glenmere primary school. It also provides explanations of the benefits of using the models and shows the links between different operations.

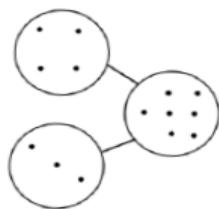
Part-Whole Model

Model examples: Addition and Subtraction



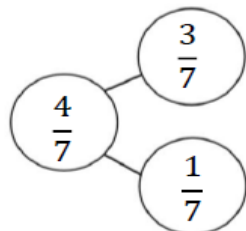
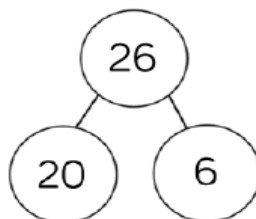
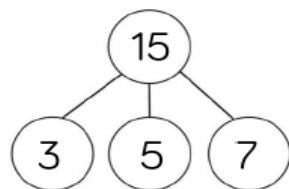
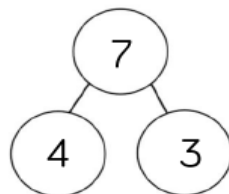
$$7 = 4 + 3$$

$$7 = 3 + 4$$



$$7 - 3 = 4$$

$$7 - 4 = 3$$



Benefits

This part-whole model supports children in their understanding of aggregation and partitioning. Due to its shape, it can be referred to as a cherry part-whole model.

When the parts are complete and the whole is empty, children use aggregation to add the parts together to find the total.

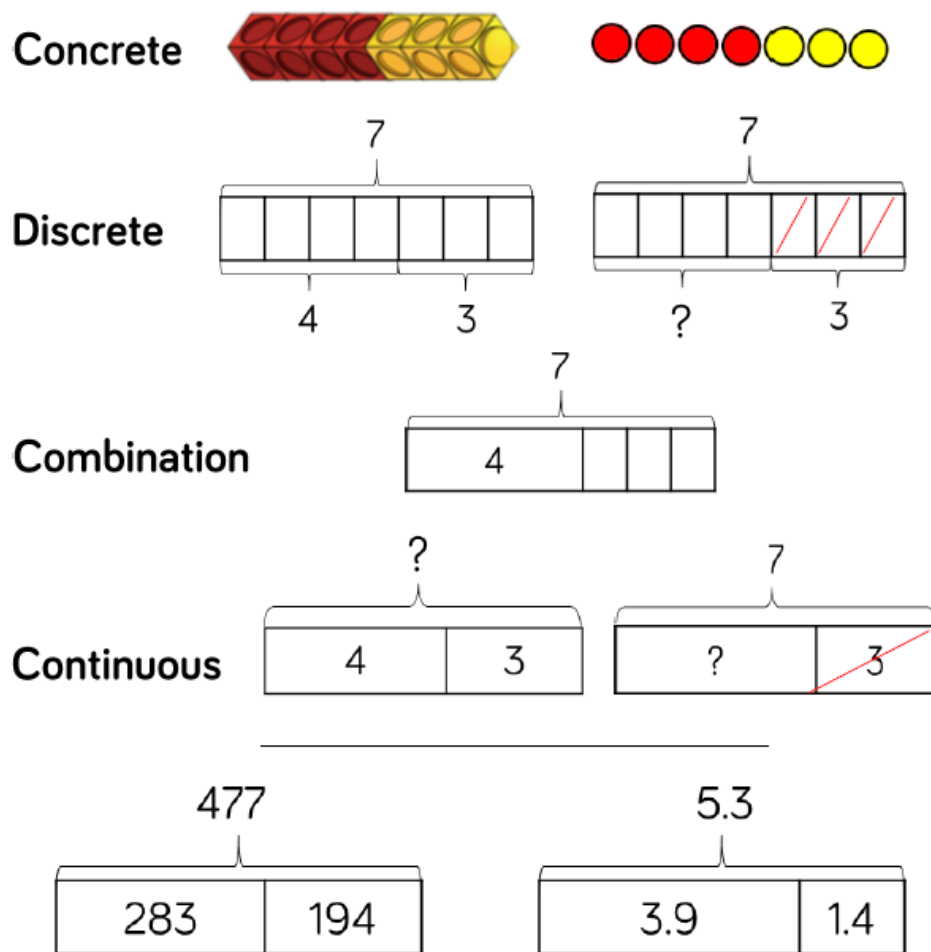
When the whole is complete and at least one of the parts is empty, children use partitioning (a form of subtraction) to find the missing part.

Part-whole models can be used to partition a number into two or more parts, or to help children to partition a number into tens and ones or other place value columns.

In KS2, children can apply their understanding of the part-whole model to add and subtract fractions, decimals and percentages.

Model examples: Addition and Subtraction

Bar Model (single)



Benefits

The single bar model is another type of a part-whole model that can support children in representing calculations to help them unpick the structure.

Cubes and counters can be used in a line as a concrete representation of the bar model.

Discrete bar models are a good starting point with smaller numbers. Each box represents one whole.

The combination bar model can support children to calculate by counting on from the larger number. It is a good stepping stone towards the continuous bar model.

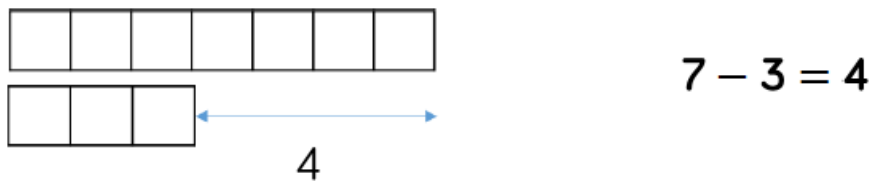
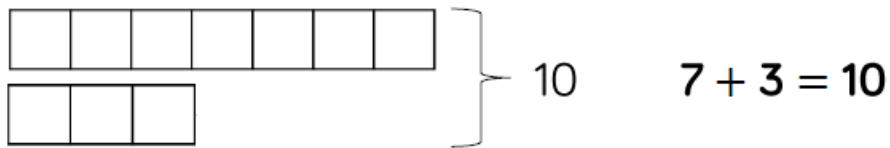
Continuous bar models are useful for a range of values. Each rectangle represents a number. The question mark indicates the value to be found.

In KS2, children can use bar models to represent larger numbers, decimals and fractions.

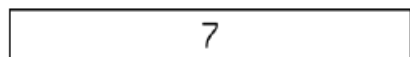
Model examples: Addition and Subtraction

Bar Model (multiple)

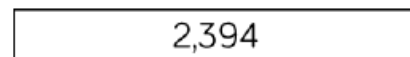
Discrete



Continuous



$$7 - 3 = 4$$



$$2,394 - 1,014 = 1,380$$

Benefits

The multiple bar model is a good way to compare quantities whilst still unpicking the structure.

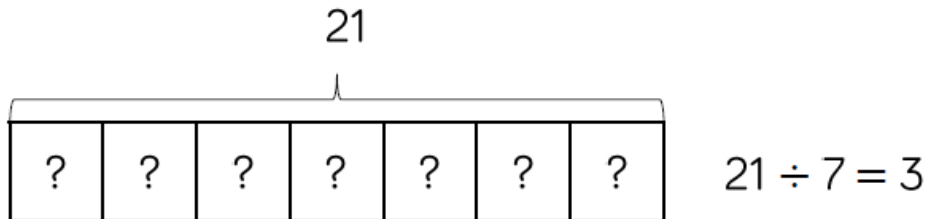
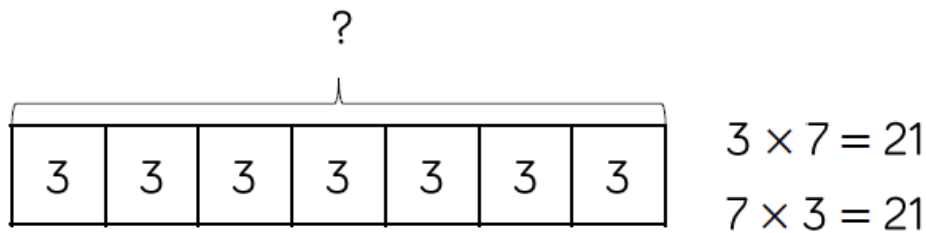
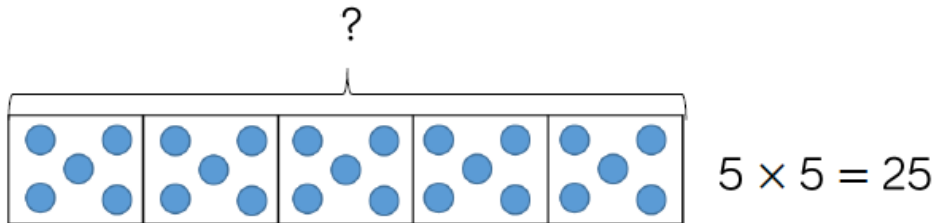
Two or more bars can be drawn, with a bracket labelling the whole positioned on the right hand side of the bars. Smaller numbers can be represented with a discrete bar model whilst continuous bar models are more effective for larger numbers.

Multiple bar models can also be used to represent the difference in subtraction. An arrow can be used to model the difference.

When working with smaller numbers, children can use cubes and a discrete model to find the difference. This supports children to see how counting on can help when finding the difference.

Model examples: Multiplication and Division

Bar Model



Benefits

Children can use the single bar model to represent multiplication as repeated addition. They could use counters, cubes or dots within the bar model to support calculation before moving on to placing digits into the bar model to represent the multiplication.

Division can be represented by showing the total of the bar model and then dividing the bar model into equal groups.

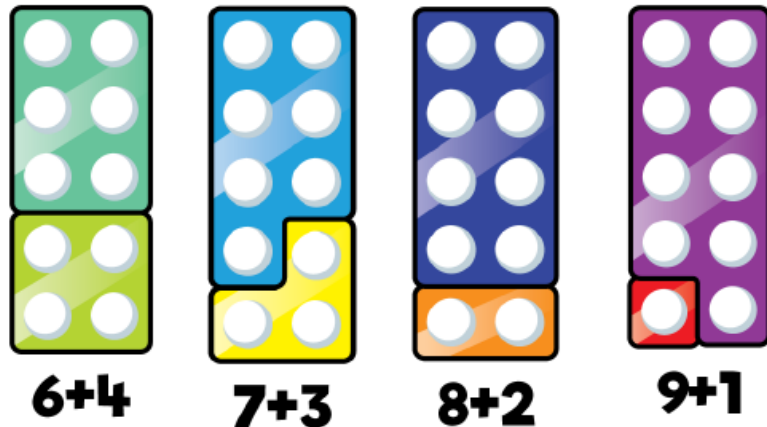
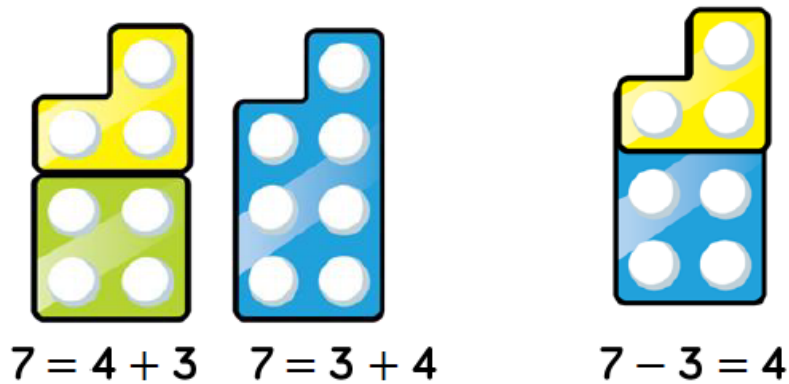
It is important when solving word problems that the bar model represents the problem.

Sometimes, children may look at scaling problems. In this case, more than one bar model is useful to represent this type of problem, e.g. There are 3 girls in a group. There are 5 times more boys than girls. How many boys are there?

The multiple bar model provides an opportunity to compare the groups.

Model examples: Addition and Subtraction

Number Shapes



Benefits

Number shapes can be useful to support children to subitise numbers as well as explore aggregation, partitioning and number bonds.

When adding numbers, children can see how the parts come together making a whole. As children use number shapes more often, they can start to subitise the total due to their familiarity with the shape of each number.

When subtracting numbers, children can start with the whole and then place one of the parts on top of the whole to see what part is missing. Again, children will start to be able to subitise the part that is missing due to their familiarity with the shapes.

Children can also work systematically to find number bonds. As they increase one number by 1, they can see that the other number decreases by 1 to find all the possible number bonds for a number.

Model examples: Multiplication and Division

Number Shapes



$$5 \times 4 = 20$$

$$4 \times 5 = 20$$

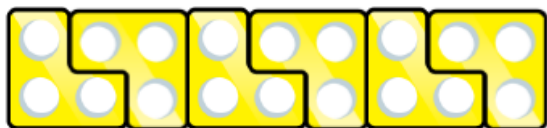


$$5 \times 4 = 20$$

$$4 \times 5 = 20$$



$$18 \div 3 = 6$$



Benefits

Number shapes support children's understanding of multiplication as repeated addition.

Children can build multiplications in a row using the number shapes. When using odd numbers, encourage children to interlock the shapes so there are no gaps in the row. They can then use the tens number shapes along with other necessary shapes over the top of the row to check the total. Using the number shapes in multiplication can support children in discovering patterns of multiplication e.g. odd \times odd = even, odd \times even = odd, even \times even = even.

When dividing, number shapes support children's understanding of division as grouping. Children make the number they are dividing and then place the number shape they are dividing by over the top of the number to find how many groups of the number there are altogether e.g. There are 6 groups of 3 in 18.

Model examples: Addition and Subtraction

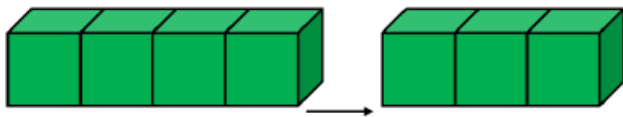
Cubes



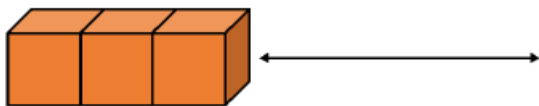
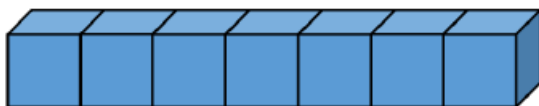
$$7 = 4 + 3$$



$$7 = 3 + 4$$



$$7 - 3 = 4$$



$$7 - 3 = 4$$

Benefits

Cubes can be useful to support children with the addition and subtraction of one-digit numbers.

When adding numbers, children can see how the parts come together to make a whole. Children could use two different colours of cubes to represent the numbers before putting them together to create the whole.

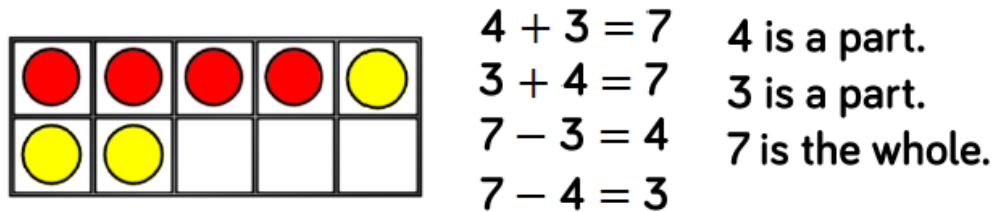
When subtracting numbers, children can start with the whole and then remove the number of cubes that they are subtracting in order to find the answer. This model of subtraction is reduction, or take away.

Cubes can also be useful to look at subtraction as difference. Here, both numbers are made and then lined up to find the difference between the numbers.

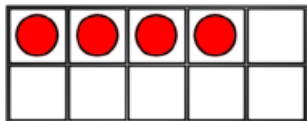
Cubes are useful when working with smaller numbers but are less efficient with larger numbers as they are difficult to subitise and children may miscount them.

Model examples: Addition and Subtraction

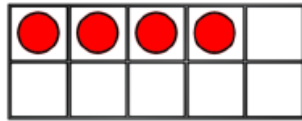
Ten Frames (within 10)



First

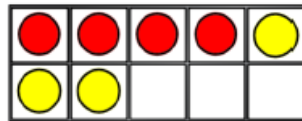


Then

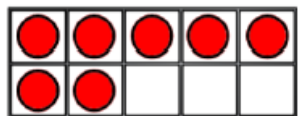


$$4 + 3 = 7$$

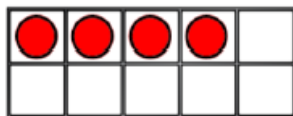
Now



First

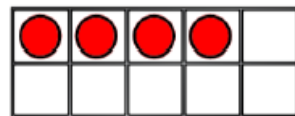


Then



$$7 - 3 = 4$$

Now



Benefits

When adding and subtracting within 10, the ten frame can support children to understand the different structures of addition and subtraction.

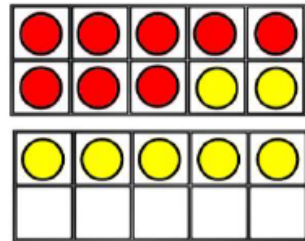
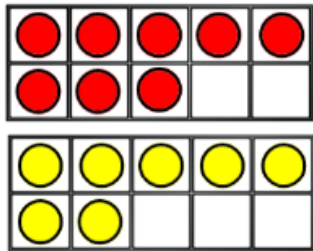
Using the language of parts and wholes represented by objects on the ten frame introduces children to aggregation and partitioning.

Aggregation is a form of addition where parts are combined together to make a whole. Partitioning is a form of subtraction where the whole is split into parts. Using these structures, the ten frame can enable children to find all the number bonds for a number.

Children can also use ten frames to look at augmentation (increasing a number) and take-away (decreasing a number). This can be introduced through a first, then, now structure which shows the change in the number in the 'then' stage. This can be put into a story structure to help children understand the change e.g. First, there were 7 cars. Then, 3 cars left. Now, there are 4 cars.

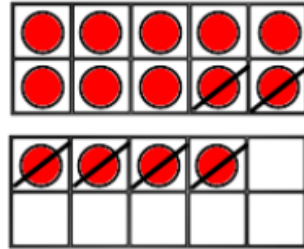
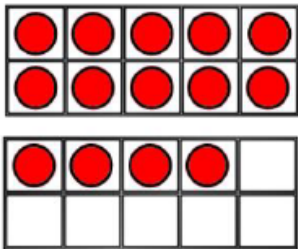
Model examples: Addition and Subtraction

Ten Frames (within 20)



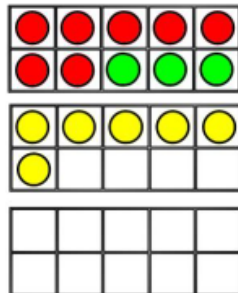
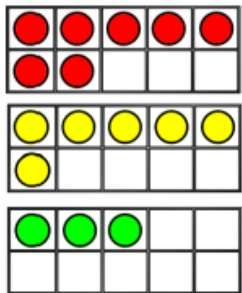
$$8 + 7 = 15$$

Diagram showing 8 partitioned into 2 and 5.



$$14 - 6 = 8$$

Diagram showing 14 partitioned into 4 and 2.



$$7 + 6 + 3 = 16$$

Diagram showing 10 as a sum of 7 and 3.

Benefits

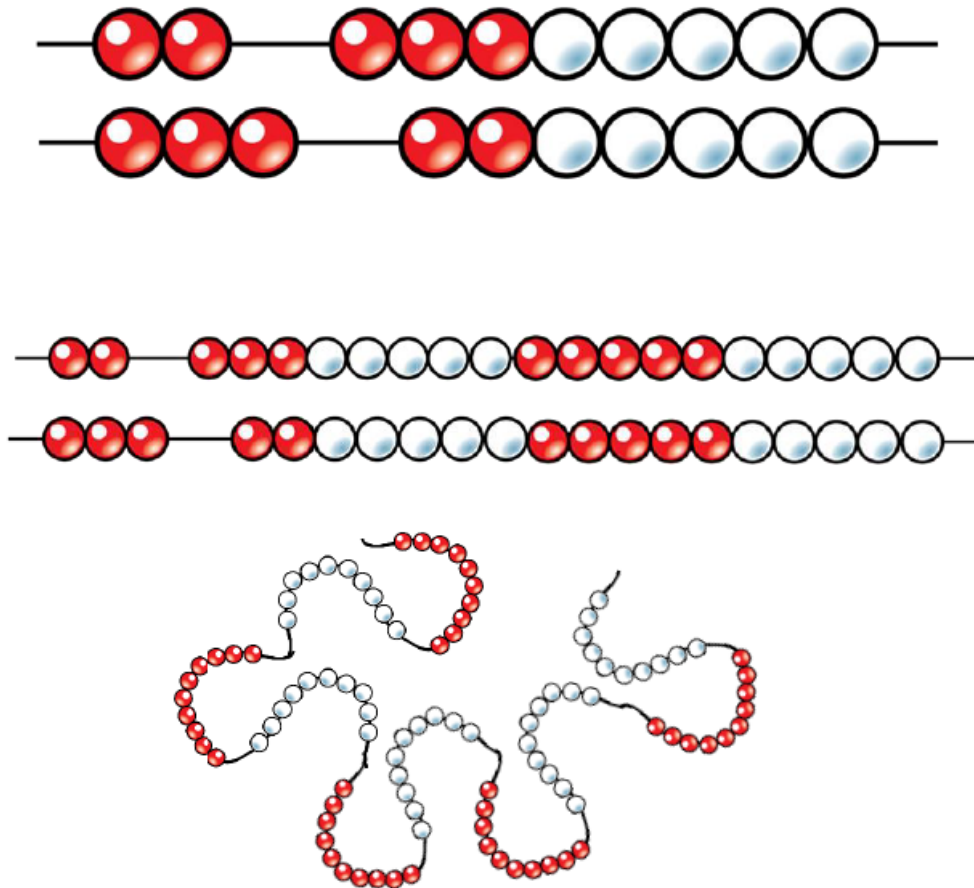
When adding two single digits, children can make each number on separate ten frames before moving part of one number to make 10 on one of the ten frames. This supports children to see how they have partitioned one of the numbers to make 10, and makes links to effective mental methods of addition.

When subtracting a one-digit number from a two-digit number, firstly make the larger number on 2 ten frames. Remove the smaller number, thinking carefully about how you have partitioned the number to make 10, this supports mental methods of subtraction.

When adding three single-digit numbers, children can make each number on 3 separate 10 frames before considering which order to add the numbers in. They may be able to find a number bond to 10 which makes the calculation easier. Once again, the ten frames support the link to effective mental methods of addition as well as the importance of commutativity.

Model examples: Addition and Subtraction

Bead Strings



Benefits

Different sizes of bead strings can support children at different stages of addition and subtraction.

Bead strings to 10 are very effective at helping children to investigate number bonds up to 10.

They can help children to systematically find all the number bonds to 10 by moving one bead at a time to see the different numbers they have partitioned the 10 beads into e.g. $2 + 8 = 10$, move one bead, $3 + 7 = 10$.

Bead strings to 20 work in a similar way but they also group the beads in fives. Children can apply their knowledge of number bonds to 10 and see the links to number bonds to 20.

Bead strings to 100 are grouped in tens and can support children in number bonds to 100 as well as helping when adding by making ten. Bead strings can show a link to adding to the next 10 on number lines which supports a mental method of addition.

Model examples: Multiplication and Division

Bead Strings



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 3 = 5$$



$$5 \times 3 = 15$$
$$3 \times 5 = 15$$

$$15 \div 5 = 3$$



$$4 \times 5 = 20$$
$$5 \times 4 = 20$$

$$20 \div 4 = 5$$

Benefits

Bead strings to 100 can support children in their understanding of multiplication as repeated addition. Children can build the multiplication using the beads. The colour of beads supports children in seeing how many groups of 10 they have, to calculate the total more efficiently.

Encourage children to count in multiples as they build the number e.g. 4, 8, 12, 16, 20.

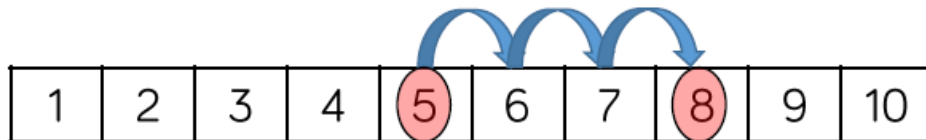
Children can also use the bead string to count forwards and backwards in multiples, moving the beads as they count.

When dividing, children build the number they are dividing and then group the beads into the number they are dividing by e.g. 20 divided by 4 - Make 20 and then group the beads into groups of four. Count how many groups you have made to find the answer.

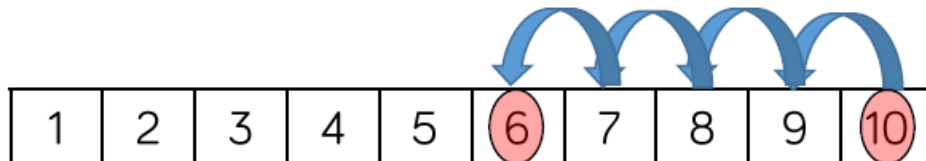
Model examples: Addition and Subtraction

Number Tracks

$$5 + 3 = 8$$



$$10 - 4 = 6$$



$$8 + 7 = 15$$



Benefits

Number tracks are useful to support children in their understanding of augmentation and reduction.

When adding, children count on to find the total of the numbers. On a number track, children can place a counter on the starting number and then count on to find the total.

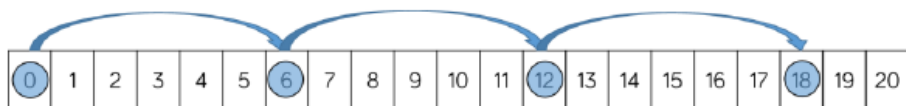
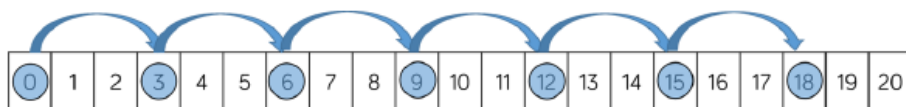
When subtracting, children count back to find their answer. They start at the minuend and then take away the subtrahend to find the difference between the numbers.

Number tracks can work well alongside ten frames and bead strings which can also model counting on or counting back.

Playing board games can help children to become familiar with the idea of counting on using a number track before they move on to number lines.

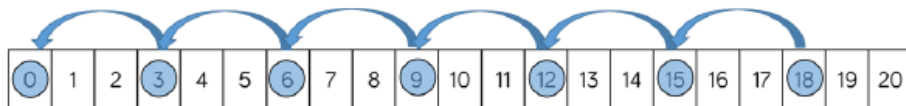
Model examples: Multiplication and Division

Number Tracks



$$6 \times 3 = 18$$

$$3 \times 6 = 18$$



$$18 \div 3 = 6$$

Benefits

Number tracks are useful to support children to count in multiples, forwards and backwards. Moving counters or cubes along the number track can support children to keep track of their counting. Translucent counters help children to see the number they have landed on whilst counting.

When multiplying, children place their counter on 0 to start and then count on to find the product of the numbers.

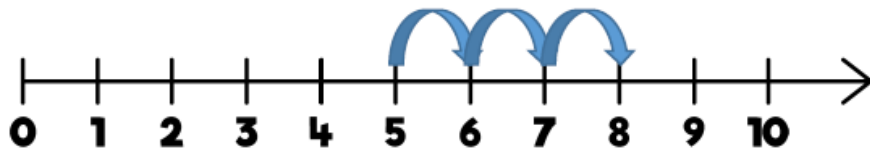
When dividing, children place their counter on the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0. Children record how many jumps they have made to find the answer to the division.

Number tracks can be useful with smaller multiples but when reaching larger numbers they can become less efficient.

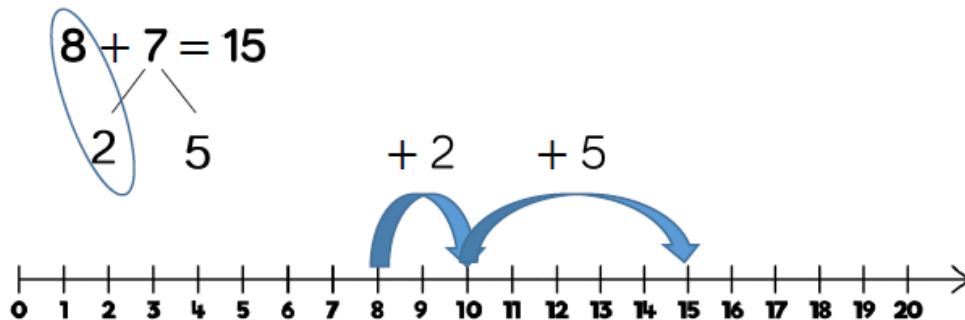
Model examples: Addition and Subtraction

Number Lines (labelled)

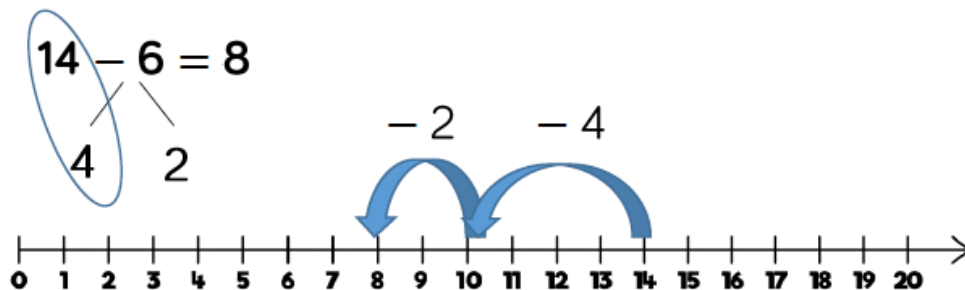
$$5 + 3 = 8$$



$$8 + 7 = 15$$



$$14 - 6 = 8$$



Benefits

Labelled number lines support children in their understanding of addition and subtraction as augmentation and reduction.

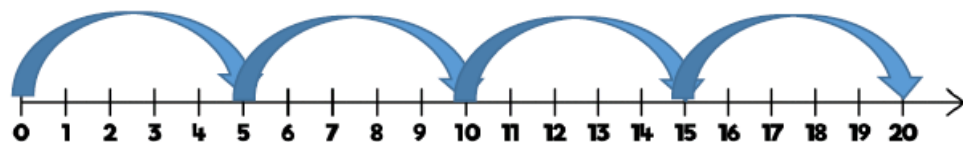
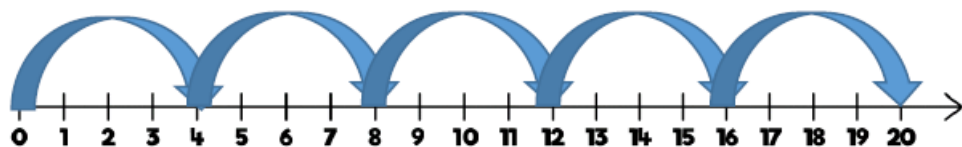
Children can start by counting on or back in ones, up or down the number line. This skill links directly to the use of the number track.

Progressing further, children can add numbers by jumping to the nearest 10 and then jumping to the total. This links to the making 10 method which can also be supported by ten frames. The smaller number is partitioned to support children to make a number bond to 10 and to then add on the remaining part.

Children can subtract numbers by firstly jumping to the nearest 10. Again, this can be supported by ten frames so children can see how they partition the smaller number into the two separate jumps.

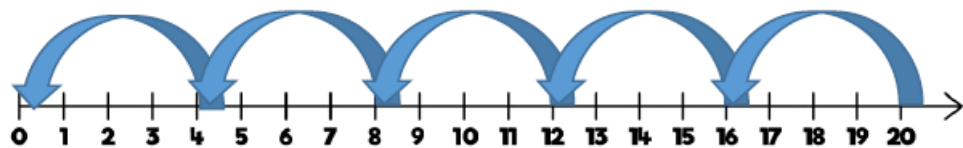
Model examples: Multiplication and Division

Number Lines (labelled)



$$4 \times 5 = 20$$

$$5 \times 4 = 20$$



$$20 \div 4 = 5$$

Benefits

Labelled number lines are useful to support children to count in multiples, forwards and backwards as well as calculating single-digit multiplications.

When multiplying, children start at 0 and then count on to find the product of the numbers.

When dividing, start at the number they are dividing and the count back in jumps of the number they are dividing by until they reach 0.

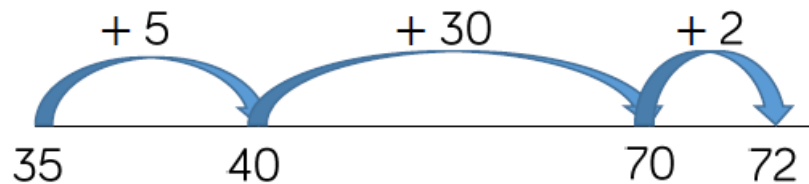
Children record how many jumps they have made to find the answer to the division.

Labelled number lines can be useful with smaller multiples, however they become inefficient as numbers become larger due to the required size of the number line.

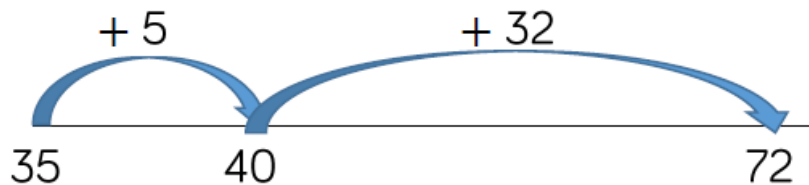
Model examples: Addition and Subtraction

Number Lines (blank)

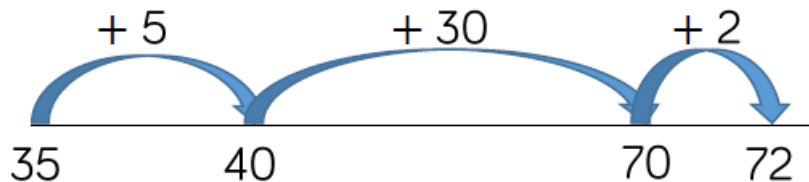
$$35 + 37 = 72$$



$$35 + 37 = 72$$



$$72 - 35 = 37$$



Benefits

Blank number lines provide children with a structure to add and subtract numbers in smaller parts.

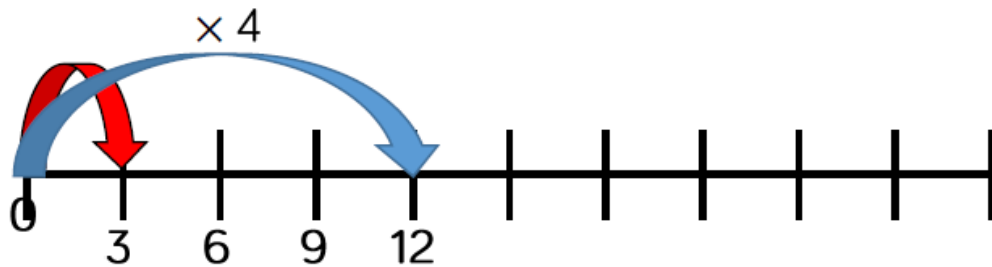
Developing from labelled number lines, children can add by jumping to the nearest 10 and then adding the rest of the number either as a whole or by adding the tens and ones separately.

Children may also count back on a number line to subtract, again by jumping to the nearest 10 and then subtracting the rest of the number.

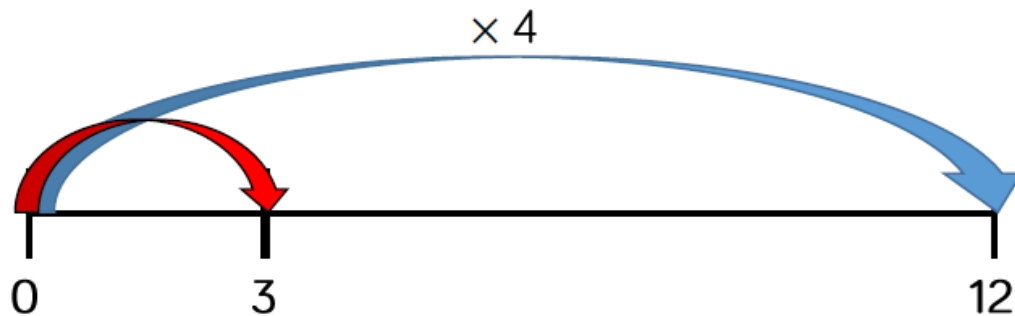
Blank number lines can also be used effectively to help children subtract by finding the difference between numbers. This can be done by starting with the smaller number and then counting on to the larger number. They then add up the parts they have counted on to find the difference between the numbers.

Model examples: Multiplication and Division

Number Lines (blank)



A red car travels 3 miles.
A blue car 4 times further.
How far does the blue car travel?



A blue car travels 12 miles.
A red car 4 times less.
How far does the red car travel?

Benefits

Children can use blank number lines to represent scaling as multiplication or division.

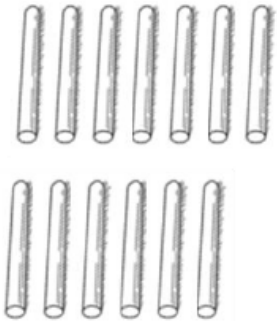
Blank number lines with intervals can support children to represent scaling accurately. Children can label intervals with multiples to calculate scaling problems.

Blank number lines without intervals can also be used for children to represent scaling.

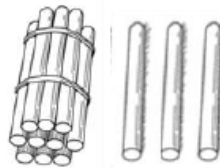
Model examples: Addition and Subtraction

Straws

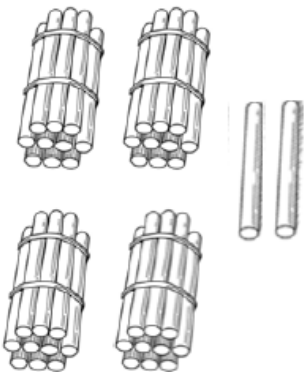
$$7 + 6 = 13$$



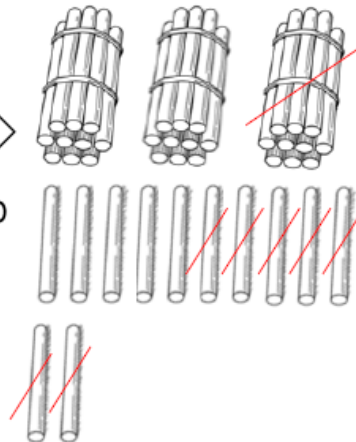
bundle together
groups of 10



$$42 - 17 = 25$$



unbundle group
of 10 straws



Benefits

Straws are an effective way to support children in their understanding of exchange when adding and subtracting 2-digit numbers.

Children can be introduced to the idea of bundling groups of ten when adding smaller numbers and when representing 2-digit numbers. Use elastic bands or other ties to make bundles of ten straws.

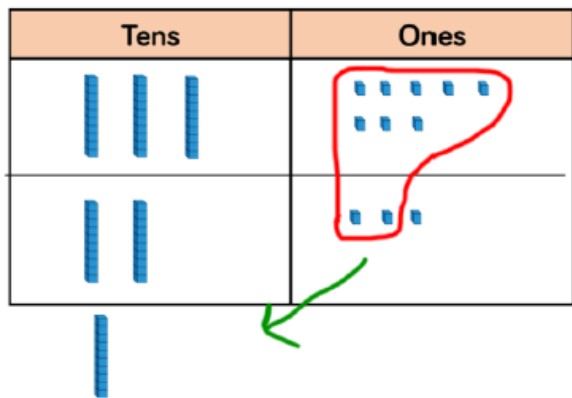
When adding numbers, children bundle a group of 10 straws to represent the exchange from 10 ones to 1 ten. They then add the individual straws (ones) and bundles of straws (tens) to find the total.

When subtracting numbers, children unbundle a group of 10 straws to represent the exchange from 1 ten to 10 ones.

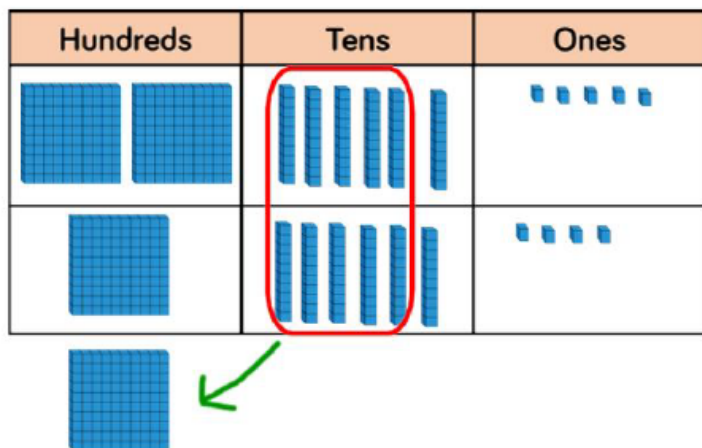
Straws provide a good stepping stone to adding and subtracting with Base 10/Dienes.

Model examples: Addition and Subtraction

Base 10/Dienes (addition)



$$\begin{array}{r}
 38 \\
 + 23 \\
 \hline
 61 \\
 \hline
 1
 \end{array}$$



$$\begin{array}{r}
 265 \\
 + 164 \\
 \hline
 429 \\
 \hline
 1
 \end{array}$$

Benefits

Using Base 10 or Dienes is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

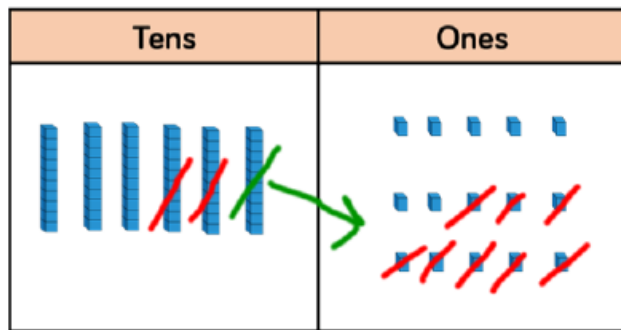
Children should first add without an exchange before moving on to addition with exchange. The representation becomes less efficient with larger numbers due to the size of Base 10. In this case, place value counters may be the better model to use.

When adding, always start with the smallest place value column. Here are some questions to support children.

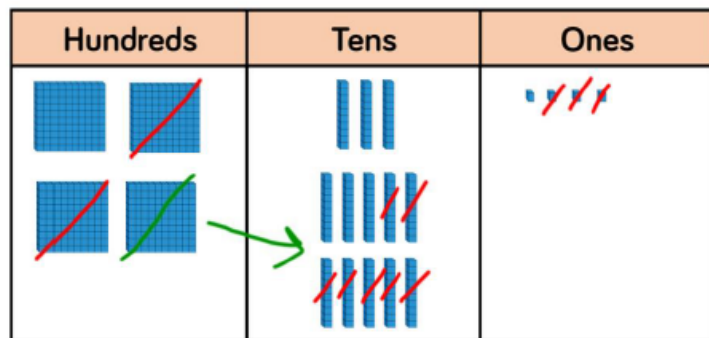
- How many ones are there altogether?
- Can we make an exchange? (Yes or No)
- How many do we exchange? (10 ones for 1 ten, show exchanged 10 in tens column by writing 1 in column)
- How many ones do we have left? (Write in ones column)
- Repeat for each column.

Model examples: Addition and Subtraction

Base 10/Dienes (subtraction)



$$\begin{array}{r}
 \overset{5}{\cancel{6}} \overset{1}{5} \\
 - 28 \\
 \hline
 37
 \end{array}$$



$$\begin{array}{r}
 \overset{3}{\cancel{4}} \overset{1}{3} 5 \\
 - 273 \\
 \hline
 262
 \end{array}$$

Benefits

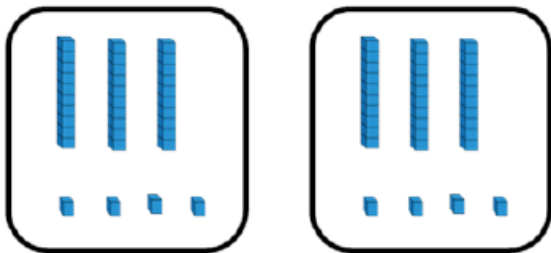
Using Base 10 or Dienes is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing Base 10 so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. When building the model, children should just make the minuend using Base 10, they then subtract the subtrahend. Highlight this difference to addition to avoid errors by making both numbers. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

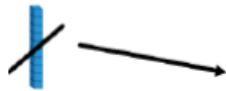
This model is efficient with up to 4-digit numbers. Place value counters are more efficient with larger numbers and decimals.

Model examples: Multiplication and Division

Base 10/Dienes (division)

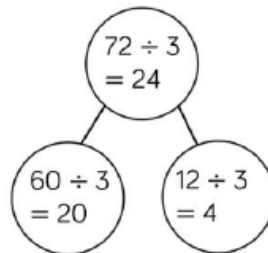


$$68 \div 2 = 34$$



Tens	Ones

$$72 \div 3 = 24$$



Benefits

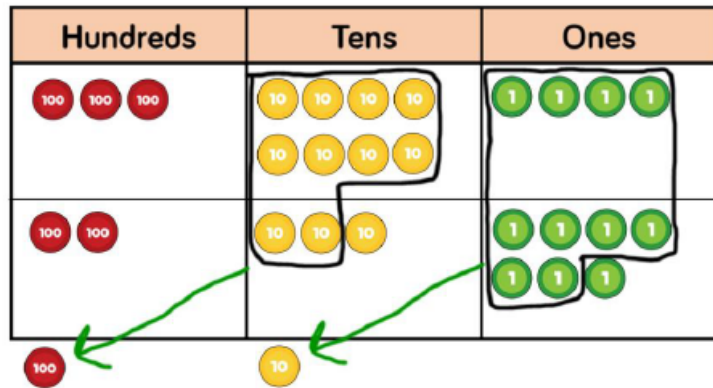
Using Base 10 or Dienes is an effective way to support children's understanding of division.

When numbers become larger, it can be an effective way to move children from representing numbers as ones towards representing them as tens and ones in order to divide. Children can then share the Base 10/ Dienes between different groups e.g. by drawing circles or by rows on a place value grid.

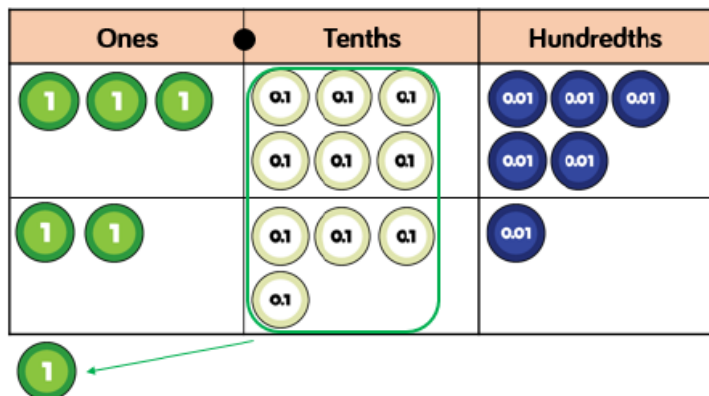
When they are sharing, children start with the larger place value and work from left to right. If there are any left in a column, they exchange e.g. one ten for ten ones. When recording, encourage children to use the part-whole model so they can consider how the number has been partitioned in order to divide. This will support them with mental methods.

Model examples: Addition and Subtraction

Place Value Counters (addition)



$$\begin{array}{r}
 384 \\
 + 237 \\
 \hline
 621 \\
 1\ 1
 \end{array}$$



$$\begin{array}{r}
 3.65 \\
 + 2.41 \\
 \hline
 6.06 \\
 1
 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column addition. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first add without an exchange before moving on to addition with exchange. Different place value counters can be used to represent larger numbers or decimals. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

When adding money, children can also use coins to support their understanding. It is important that children consider how the coins link to the written calculation especially when adding decimal amounts.

Model examples: Addition and Subtraction

Place Value Counters (Subtraction)

Hundreds	Tens	Ones

$$\begin{array}{r}
 \overset{4}{\cancel{6}}\overset{1}{\cancel{5}}2 \\
 - 207 \\
 \hline
 445
 \end{array}$$

Thousands	Hundreds	Tens	Ones

$$\begin{array}{r}
 \overset{3}{\cancel{4}}\overset{1}{\cancel{3}}57 \\
 - 2735 \\
 \hline
 1622
 \end{array}$$

Benefits

Using place value counters is an effective way to support children's understanding of column subtraction. It is important that children write out their calculations alongside using or drawing counters so they can see the clear links between the written method and the model.

Children should first subtract without an exchange before moving on to subtraction with exchange. If you don't have place value counters, use normal counters on a place value grid to enable children to experience the exchange between columns.

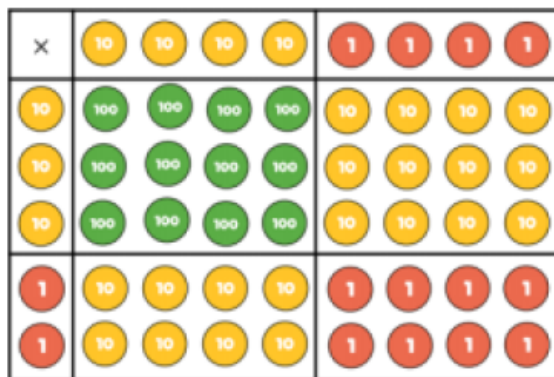
When building the model, children should just make the minuend using counters, they then subtract the subtrahend. Children start with the smallest place value column. When there are not enough ones/tens/hundreds to subtract in a column, children need to move to the column to the left and exchange e.g. exchange 1 ten for 10 ones. They can then subtract efficiently.

Model examples: Multiplication and Division

Place Value Counters (multiplication)



$$\begin{array}{r}
 44 \\
 \times 5 \\
 \hline
 220 \\
 120 \\
 \hline
 220
 \end{array}$$



$$\begin{array}{r}
 44 \\
 \times 32 \\
 \hline
 88 \\
 880 \\
 + 1200 \\
 \hline
 1408 \\
 1
 \end{array}$$

Benefits

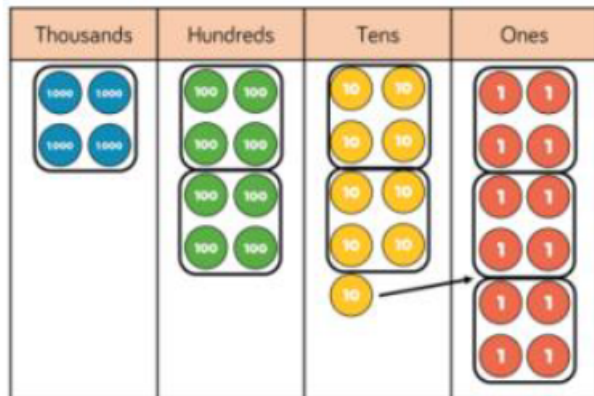
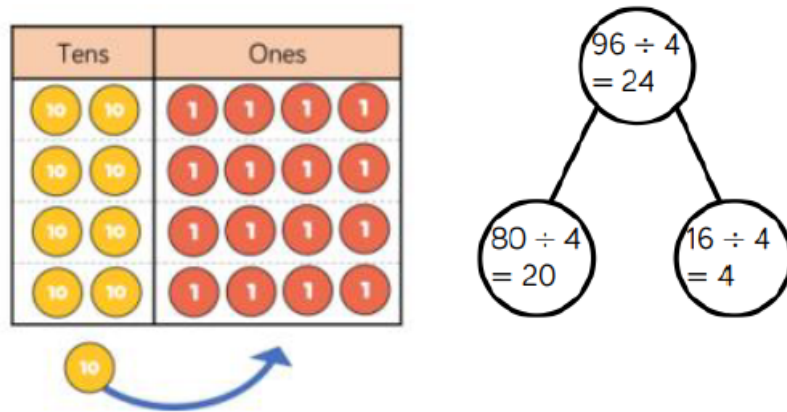
Using place value counters is an effective way to support children's understanding of column multiplication. It is important that children write out their calculation alongside the equipment so they can see how the concrete and written match.

As numbers become larger in multiplication or the amounts of groups becomes higher, Base 10 / Dienes becomes less efficient due to the amount of equipment and number of exchanges needed. The counters should be used to support the understanding of the written method rather than support the arithmetic.

Place value counters also support the area model of multiplication well. Children can see how to multiply 2-digit numbers by 2-digit numbers.

Model examples: Multiplication and Division

Place Value Counters (division)



$$4 \overline{) 1223}$$

Benefits

Using place value counters is an effective way to support children's understanding of division.

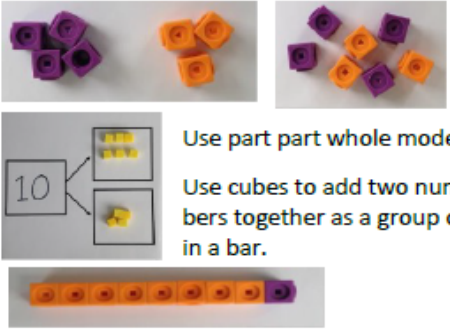
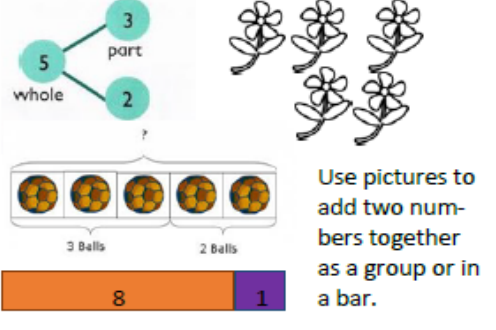


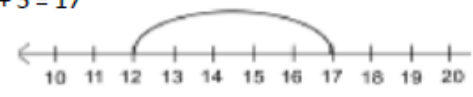
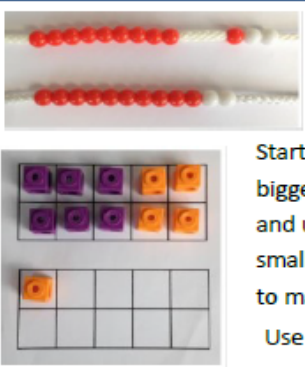
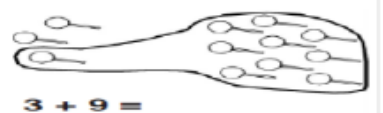
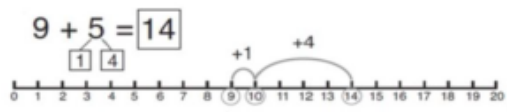

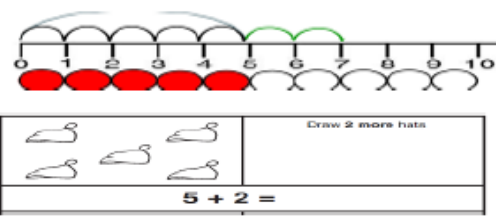
When working with smaller numbers, children can use place value counters to share between groups. They start by sharing the larger place value column and work from left to right. If there are any counters left over once they have been shared, they exchange the counter e.g. exchange one ten for ten ones. This method can be linked to the part-whole model to support children to show their thinking.

Place value counters also support children's understanding of short division by grouping the counters rather than sharing them. Children work from left to right through the place value columns and group the counters in the number they are dividing by. If there are any counters left over after they have been grouped, they exchange the counter e.g. exchange one hundred for ten tens.

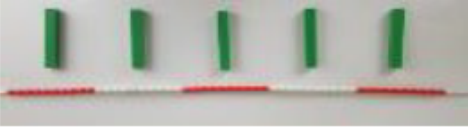

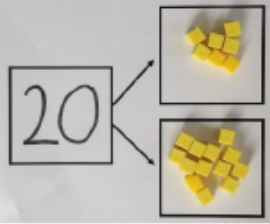
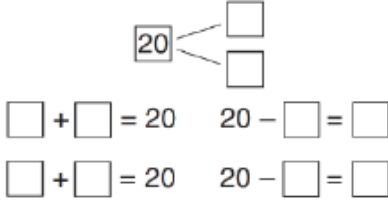
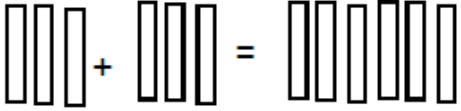
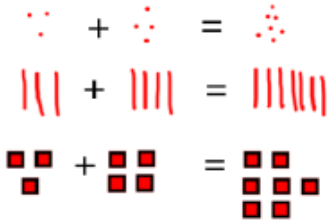


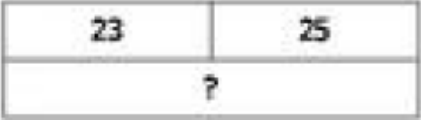
The next part of this policy is an overview of the skills linked to each year group to support consistency through out the school. A glossary of key Mathematical vocabulary is provided at the end of the calculation policy to support understanding of the key language used to teach the four operations.



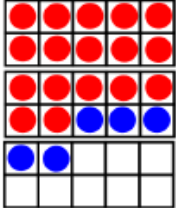
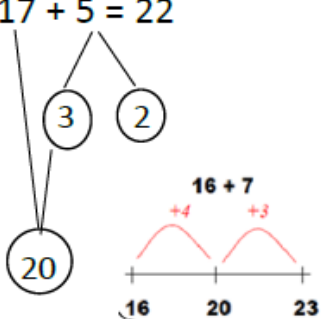

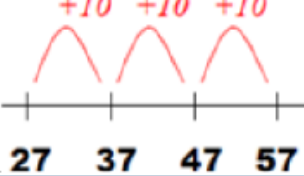

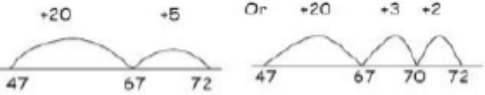
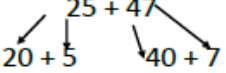


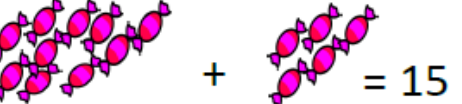
Year 1 - Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model	 <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	$4 + 3 = 7$  <p>Use the part-part whole diagram as shown above to move into the abstract.</p> $10 = 6 + 4$
Starting at the bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	$12 + 5 = 17$  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	$5 + 12 = 17$ <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
Regrouping to make 10. <i>This is an essential skill for column addition later.</i>	 <p>$6 + 5 = 11$</p> <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames.</p>	 <p>$3 + 9 =$</p> <p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.</p> $9 + 5 = 14$ 	$7 + 4 = 11$ <p>If I am at seven, how many more do I need to make 10. How many more do I add on now?</p>
Represent & use number bonds and related subtraction facts within 20	 <p>2 more than 5.</p>	 <p>$5 + 2 =$</p>	<p>Emphasis should be on the language</p> <p>'1 more than 5 is equal to 6.'</p> <p>'2 more than 5 is 7.'</p> <p>'8 is 3 more than 5.'</p>

Year 2 - Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \text{---} \text{ tens}$ $30 + 50 = \text{---}$ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts <i>Part part whole</i>	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts	$\square\square + \square\square = \square\square\square\square$ 	 Children draw representations of H,T and O	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$

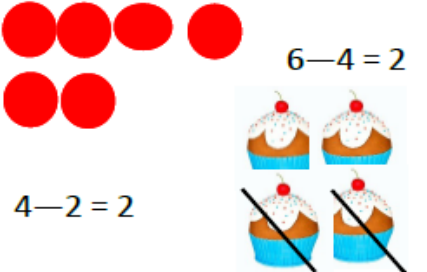
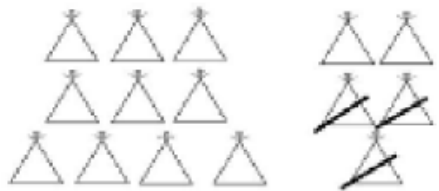
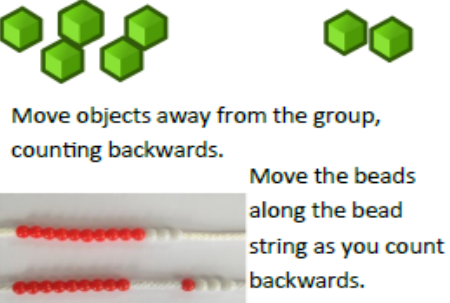
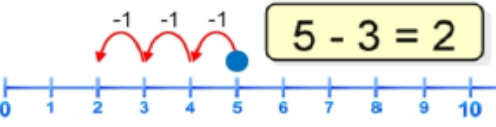
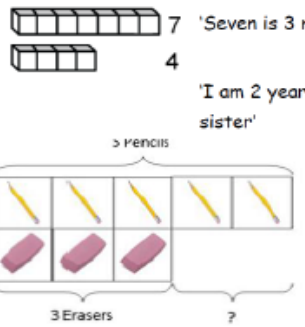
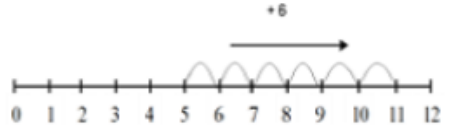
Year 2 - Addition continued

Objective & Strategy	Concrete	Pictorial	Abstract				
<p>Add a two digit number and ones</p>	 <p style="text-align: right;">$17 + 5 = 22$</p> <p style="text-align: right;">Use ten frame to make 'magic ten'</p> <p>Children explore the pattern.</p> <p>$17 + 5 = 22$</p> <p>$27 + 5 = 32$</p>	<p>Use part part whole and number line to model.</p> <p style="text-align: center;">$17 + 5 = 22$</p> 	<p>$17 + 5 = 22$</p> <p>Explore related facts</p> <p>$17 + 5 = 22$</p> <p>$5 + 17 = 22$</p> <p>$22 - 17 = 5$</p> <p>$22 - 5 = 17$</p> <table border="1" style="margin-left: auto; margin-right: auto; text-align: center;"> <tr><td colspan="2">22</td></tr> <tr><td>17</td><td>5</td></tr> </table>	22		17	5
22							
17	5						
<p>Add a 2 digit number and tens</p>	 <p>$25 + 10 = 35$</p> <p>Explore that the ones digit does not change</p>	<p style="text-align: center;">$27 + 30$</p> 	<p>$27 + 10 = 37$</p> <p>$27 + 20 = 47$</p> <p>$27 + \square = 57$</p>				
<p>Add two 2-digit numbers</p>	 <p>Model using dienes, place value counters and numicon</p>	 <p>Use number line and bridge ten using part whole if necessary.</p>	<p style="text-align: center;">$25 + 47$</p>  <p>$20 + 40 = 60$</p> <p>$5 + 7 = 12$</p> <p>$60 + 12 = 72$</p>				
<p>Add three 1-digit numbers</p>	 <p>Combine to make 10 first if possible, or bridge 10 then add third digit</p>	 <p>Regroup and draw representation.</p>  <p>$= 15$</p>	<p style="text-align: center;">$4 + 7 + 6 = 10 + 7$</p> <p style="text-align: center;">$= 17$</p> <p>Combine the two numbers that make/ bridge ten then add on the third.</p>				

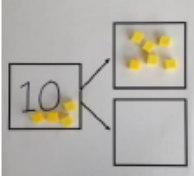
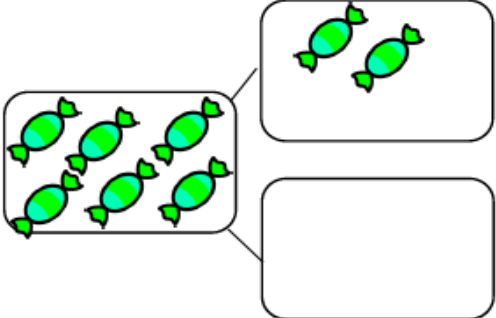
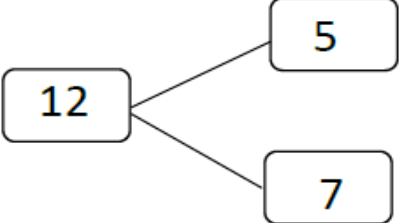

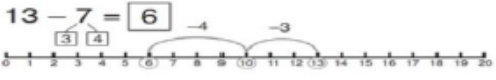


Year 3 - Addition

Objective & Strategy	Concrete	Pictorial	Abstract																							
<p>Column Addition—no regrouping (friendly numbers)</p> <p>Add two or three 2 or 3-digit numbers.</p>	<div style="border: 1px solid black; padding: 5px; margin-bottom: 10px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 50%; text-align: center;">T</td> <td style="width: 50%; text-align: center;">O</td> </tr> <tr> <td style="text-align: center;"> </td> <td style="text-align: center;"> </td> </tr> </table> </div> <p>Model using Dienes or numicon</p> <p>Add together the ones first, then the tens.</p> <table style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <tr> <td></td> <td style="text-align: center; border-bottom: 1px solid black;">Tens</td> <td style="text-align: center; border-bottom: 1px solid black;">Units</td> </tr> <tr> <td style="text-align: right;">45</td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> <tr> <td style="text-align: right;">34</td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> <tr> <td></td> <td style="text-align: center; border-top: 1px solid black;">7</td> <td style="text-align: center; border-top: 1px solid black;">9</td> </tr> </table> <div style="border: 1px solid black; padding: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 30%; text-align: center;"></td> <td style="width: 30%; text-align: center;"> $\begin{array}{r} 21 + 42 = \\ 21 \\ + 42 \\ \hline \end{array}$ </td> <td style="width: 40%;"></td> </tr> </table> </div> <p>Move to using place value counters</p>	T	O				Tens	Units	45			34				7	9		$\begin{array}{r} 21 + 42 = \\ 21 \\ + 42 \\ \hline \end{array}$		<p>Children move to drawing the counters using a tens and one frame.</p> <table style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 50%; border-right: 1px solid black; padding: 10px;">tens</td> <td style="width: 50%; padding: 10px;">ones</td> </tr> <tr> <td style="border-right: 1px solid black;"></td> <td></td> </tr> </table>	tens	ones			$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
T	O																									
	Tens	Units																								
45																										
34																										
	7	9																								
	$\begin{array}{r} 21 + 42 = \\ 21 \\ + 42 \\ \hline \end{array}$																									
tens	ones																									
<p>Column Addition with regrouping.</p>	<table style="width: 100%; border-collapse: collapse; margin-bottom: 10px;"> <tr> <td></td> <td style="text-align: center; border-bottom: 1px solid black;">Tens</td> <td style="text-align: center; border-bottom: 1px solid black;">Units</td> </tr> <tr> <td style="text-align: right;">39</td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> <tr> <td style="text-align: right;">15</td> <td style="text-align: center;"></td> <td style="text-align: center;"></td> </tr> <tr> <td></td> <td style="text-align: center; border-top: 1px solid black;">5</td> <td style="text-align: center; border-top: 1px solid black;">4</td> </tr> </table> <p>Exchange ten ones for a ten. Model using numicon and pv counters.</p> <div style="border: 1px solid black; padding: 5px;"> <table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 30%; text-align: center;"></td> <td style="width: 30%; text-align: center;"> $\begin{array}{r} 146 \\ + 527 \\ \hline \end{array}$ </td> <td style="width: 40%;"></td> </tr> </table> </div>		Tens	Units	39			15				5	4		$\begin{array}{r} 146 \\ + 527 \\ \hline \end{array}$		<table style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td style="width: 50%; border-right: 1px solid black; padding: 10px;"></td> <td style="width: 50%; padding: 10px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 10px;"></td> <td style="padding: 10px;"></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 10px;">5</td> <td style="padding: 10px;">1</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 10px;"></td> <td></td> </tr> </table> <p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</p>					5	1			$\begin{array}{r} 20 + 5 \\ 40 + 8 \\ \hline 60 + 13 = 73 \end{array}$ <p>Start by partitioning the numbers before formal column to show the exchange.</p> $\begin{array}{r} 536 \\ + 85 \\ \hline 621 \\ 11 \end{array}$
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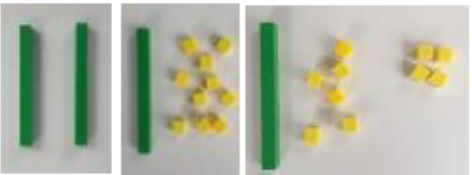


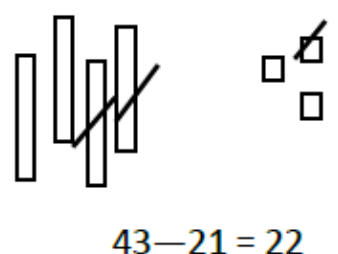
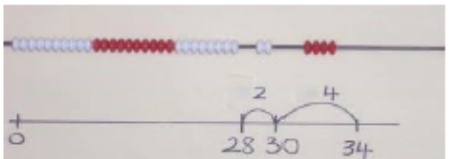
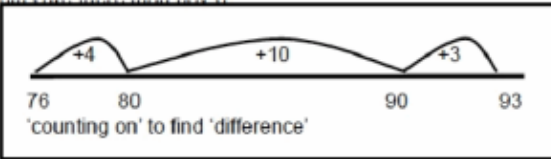
Year 1 - Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Taking away ones.</p>	<p>Use physical objects, counters, cubes etc to show how objects can be taken away.</p>  <p>$6 - 4 = 2$</p> <p>$4 - 2 = 2$</p> <p>$6 - 4 = 2$</p>	 <p>$15 - 3 = 12$</p> <p>Cross out drawn objects to show what has been taken away.</p>	<p>$7 - 4 = 3$</p> <p>$16 - 9 = 7$</p>
<p>Counting back</p>	 <p>Move objects away from the group, counting backwards.</p> <p>Move the beads along the bead string as you count backwards.</p>	 <p>$5 - 3 = 2$</p> <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p>
<p>Find the Difference</p>	<p>Compare objects and amounts</p>  <p>'Seven is 3 more than four'</p> <p>4</p> <p>'I am 2 years older than my sister'</p> <p>3 Pencils</p> <p>3 Erasers</p> <p>?</p> <p>Lay objects to represent bar model.</p>	<p>Count on using a number line to find the difference.</p>  <p>$+6$</p>	<p>Hannah has 12 sweets and her sister has 5. How many more does Hannah have than her sister.?</p>

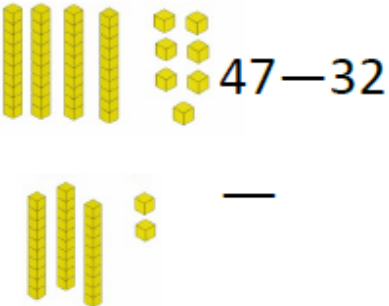
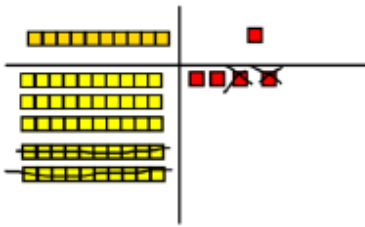
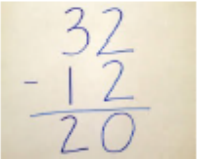
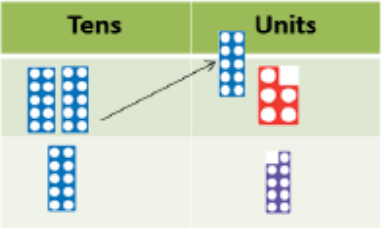
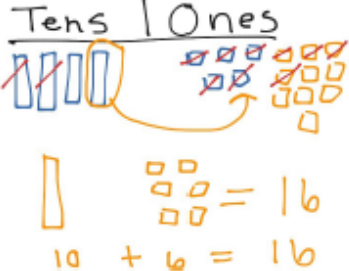
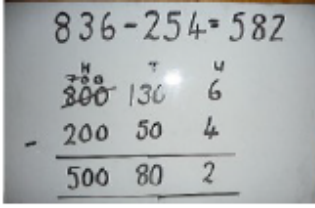
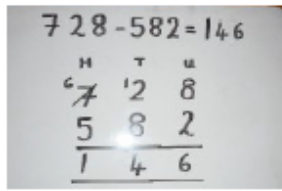
Year 1 - Subtraction continued

Objective & Strategy	Concrete	Pictorial	Abstract		
<p>Represent and use number bonds and related subtraction facts within 20</p> <p>Part Part Whole model</p>	 <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$	 <p>Use pictorial representations to show the part.</p>	<p>Move to using numbers within the part whole model.</p> 		
<p>Make 10</p>	<p style="text-align: center;">14 - 9</p>  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	<p style="text-align: center;">13 - 7</p>  <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	<p style="text-align: center;">16 - 8</p> <p>How many do we take off first to get to 10? How many left to take off?</p>		
<p>Bar model</p>	 $5 - 2 = 3$		<table border="1" style="width: 100%; text-align: center;"> <tr> <td style="width: 75%;">8</td> <td style="width: 25%;">2</td> </tr> </table> $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$	8	2
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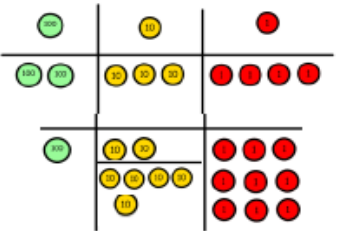
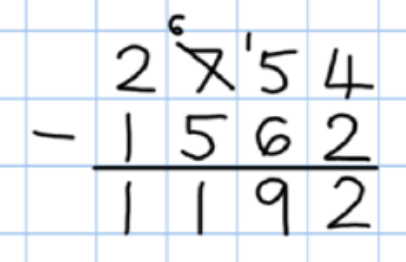
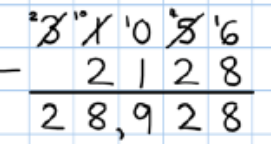
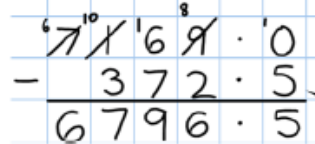
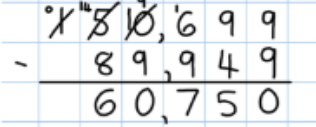
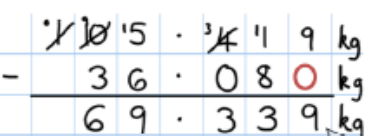
Year 2 - Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>		$20 - 4 = 16$
Partitioning to subtract without regrouping. <i>'Friendly numbers'</i>	$34 - 13 = 21$  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	Children draw representations of Dienes and cross off. 	$43 - 21 = 22$
Make ten strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	 $34 - 28$ <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	$93 - 76 = 17$

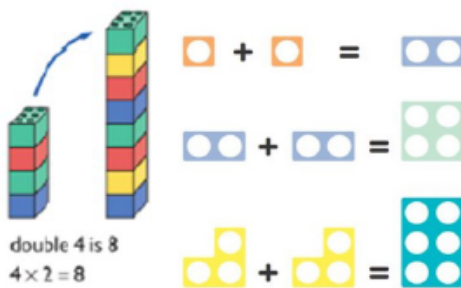

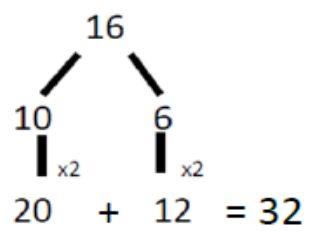
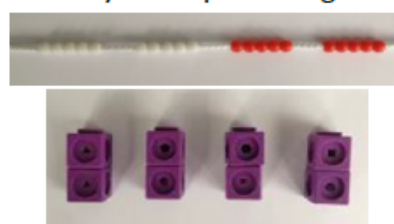
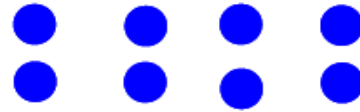
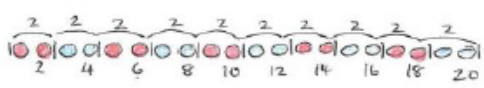
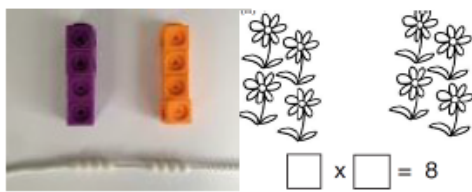

Year 3 - Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Column subtraction without regrouping (friendly numbers)</p>	 <p style="text-align: center;">$47 - 32$</p> <p style="text-align: center;">Use base 10 or Numicon to model</p>	 <p style="text-align: center;">Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ <p>Darw representations to support understanding</p>	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 
<p>Column subtraction with regrouping</p>	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p>	$\begin{array}{r} 45 \\ - 29 \\ \hline 16 \end{array}$ <p style="text-align: center;">Tens Ones</p>  <p>Children may draw base ten or PV counters and cross off.</p>	 <p>Begin by partitioning into pv columns</p>  <p>Then move to formal method.</p>

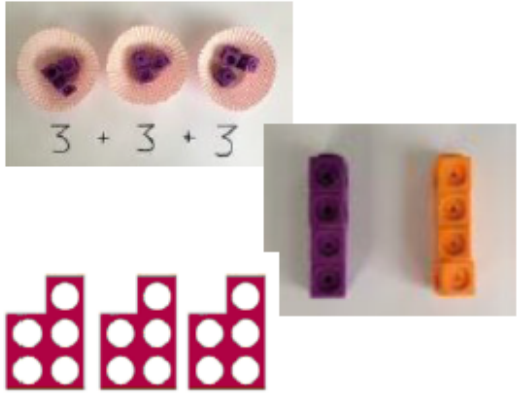
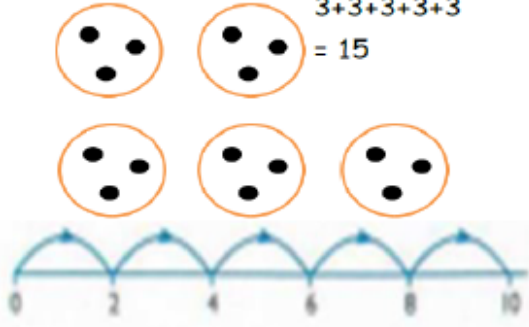

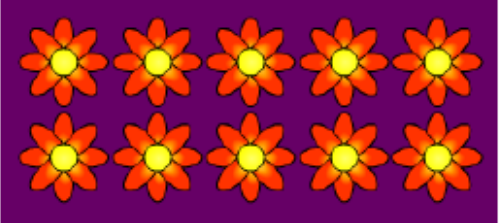
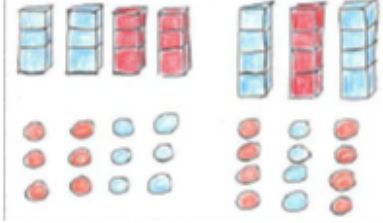
Year 4 - Year 6 - Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Subtracting tens and ones</p> <p>Year 4 subtract with up to 4 digits.</p> <p><i>Introduce decimal subtraction through context of money</i></p>	<p style="text-align: center;">$234 - 179$</p>  <p style="text-align: center;">Model process of exchange using Numicon, base ten and then move to PV counters.</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p style="text-align: center;">Use the phrase 'take and make' for exchange</p>
<p>Year 5- Subtract with at least 4 digits, including money and measures.</p> <p><i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i></p>	<p>As Year 4</p>	<p>Children to draw pv counters and show their exchange—see Y3</p>	 <p>Use zeros for place-holders.</p> 
<p>Year 6—Subtract with increasingly large and more complex numbers and decimal values.</p>			 

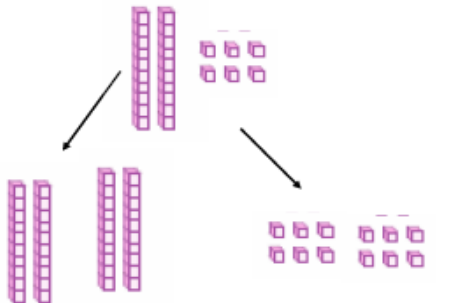
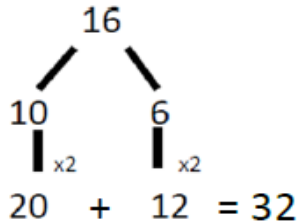
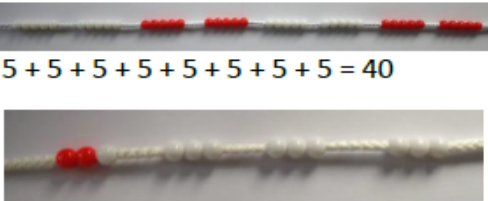

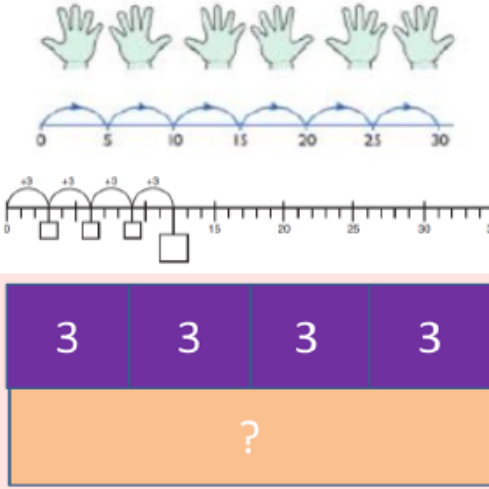
Year 1 - Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Doubling</p>	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers</p> <p style="text-align: center;">Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p> 
<p>Counting in multiples</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>
<p>Making equal groups and counting the total</p>	 <p>$\square \times \square = 8$</p> <p>Use manipulatives to create equal groups.</p>	<p>Draw  to show $2 \times 3 = 6$</p> <p>Draw and make representations</p>	<p style="text-align: center;">$2 \times 4 = 8$</p>

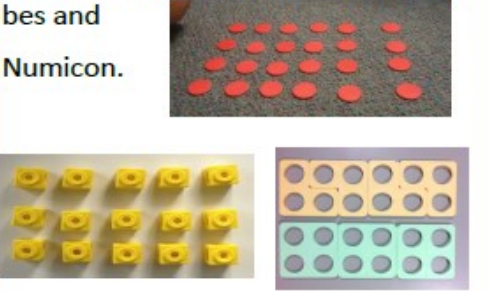
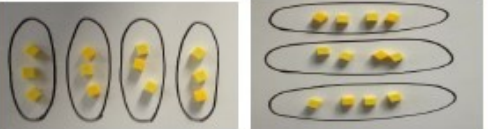
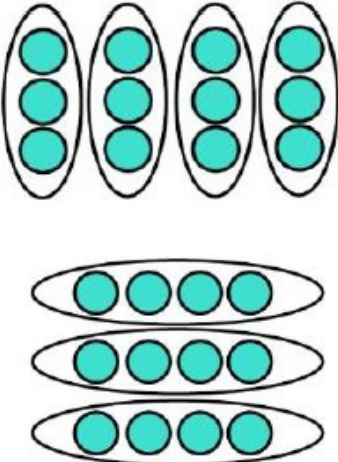


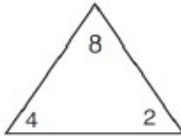
Year 1 - Multiplication continued

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Repeated addition</p>	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve prob</p> <p>There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> 	<p>Write addition sentences to describe objects and pictures.</p> 
<p>Understanding arrays</p>	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	<p>Draw representations of arrays to show understanding</p> 	<p style="text-align: center;">$3 \times 2 = 6$</p> <p style="text-align: center;">$2 \times 5 = 10$</p>

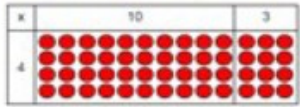
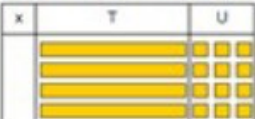

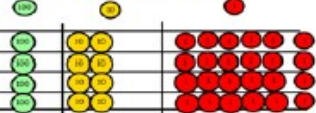
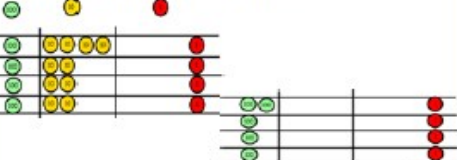
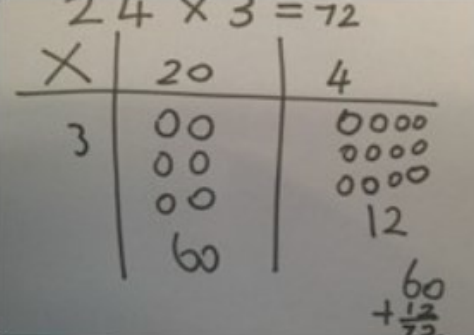
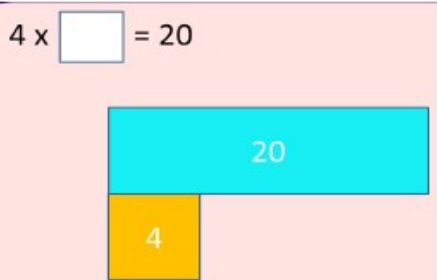
Year 2 - Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Model doubling using dienes and PV counters.</p>  <p style="text-align: center;">$40 + 12 = 52$</p>	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p>  <p style="text-align: center;">$20 + 12 = 32$</p>
<p>Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)</p>	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p>  <p style="text-align: center;">$5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$</p> 	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10</p> <p>0, 3, 6, 9, 12, 15</p> <p>0, 5, 10, 15, 20, 25, 30</p> <p style="text-align: center;">$4 \times 3 = \square$</p>



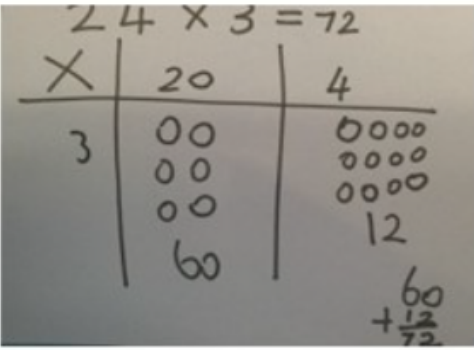
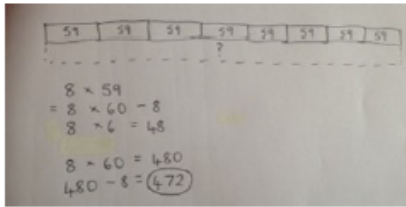
Year 2 - Multiplication continued

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Multiplication is commutative</p>	<p>Create arrays using counters and cubes and Numicon.</p>  <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p> 	<p>Use representations of arrays to show different calculations and explore commutativity.</p> 	<p>$12 = 3 \times 4$ $12 = 4 \times 3$</p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$</p>
<p>Using the Inverse</p> <p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p>		 <p><input type="text"/> \times <input type="text"/> = <input type="text"/></p> <p><input type="text"/> \times <input type="text"/> = <input type="text"/></p> <p><input type="text"/> \div <input type="text"/> = <input type="text"/></p> <p><input type="text"/> \div <input type="text"/> = <input type="text"/></p>	<p>$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$</p> <p>Show all 8 related fact family sentences.</p>

Year 3 - Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract															
<p>Grid method</p>	<p>Show the links with arrays to first introduce the grid method.</p>  <p>4 rows of 10 4 rows of 3</p> <p>Move onto base ten to move towards a more compact method.</p>  <p>4 rows of 13</p> <p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4×126</p> <p>Fill each row with 126</p>  <p>Calculations 4×126</p> <p>Add up each column, starting with the ones making any exchanges needed</p>  <p>Then you have your answer.</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p>  <p>Bar model are used to explore missing numbers</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1621 501 1935 580"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p style="text-align: center;">$210 + 35 = 245$</p> <p>Moving forward, multiply by a 2 digit number showing the different rows within the grid method.</p> <table border="1" data-bbox="1621 868 1935 1043"> <tr> <td></td> <td>10</td> <td>8</td> </tr> <tr> <td>10</td> <td>100</td> <td>80</td> </tr> <tr> <td>3</td> <td>30</td> <td>24</td> </tr> </table>	x	30	5	7	210	35		10	8	10	100	80	3	30	24
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Year 4 - Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract																																																															
<p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Calculations 4×126</p> <p>Fill each row with 126</p>  <p>Add up each col. making any exchanges needed</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1579 486 1892 582"> <tr> <td>x</td> <td>30</td> <td>5</td> </tr> <tr> <td>7</td> <td>210</td> <td>35</td> </tr> </table> <p>$210 + 35 = 245$</p>	x	30	5	7	210	35																																																									
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<p>Column multiplication</p>	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p> <table border="1" data-bbox="504 1069 817 1428"> <thead> <tr> <th>Hundreds</th> <th>Tens</th> <th>Ones</th> </tr> </thead> <tbody> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </tbody> </table> <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding long multiplication is modelled alongside</p>	Hundreds	Tens	Ones													<table border="1" data-bbox="1075 965 1400 1045"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table> <p>The grid method may be used to show how this relates to a formal written method.</p>  <p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>	x	300	20	7	4	1200	80	28	<table style="margin-left: auto; margin-right: auto;"> <tr><td></td><td></td><td></td><td>327</td></tr> <tr><td></td><td>x</td><td></td><td>4</td></tr> <tr><td></td><td></td><td></td><td>28</td></tr> <tr><td></td><td></td><td></td><td>80</td></tr> <tr><td></td><td></td><td></td><td>1200</td></tr> <tr><td></td><td></td><td></td><td>1308</td></tr> </table> <p>This may lead to a compact method.</p> <table border="1" data-bbox="1534 1316 1758 1492"> <tr> <td></td> <td>3</td> <td>2</td> <td>7</td> </tr> <tr> <td>x</td> <td></td> <td></td> <td>4</td> </tr> <tr> <td></td> <td>1</td> <td>3</td> <td>0</td> </tr> <tr> <td></td> <td></td> <td>1</td> <td>2</td> </tr> </table>				327		x		4				28				80				1200				1308		3	2	7	x			4		1	3	0			1	2
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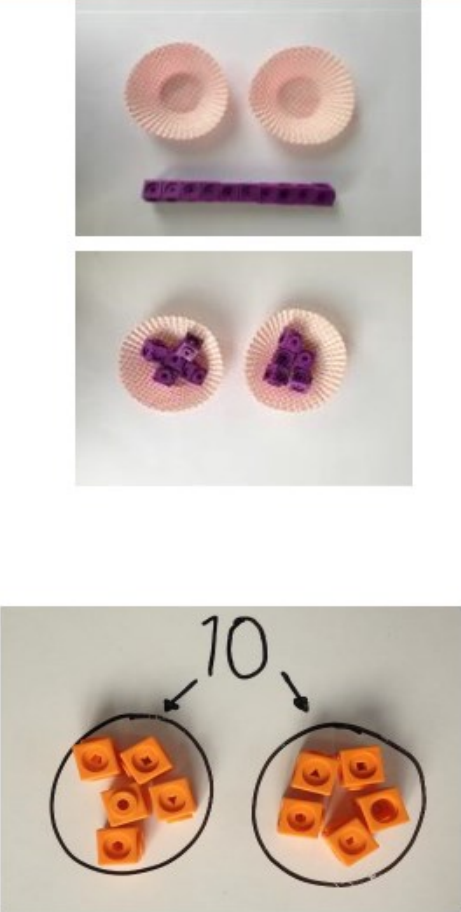
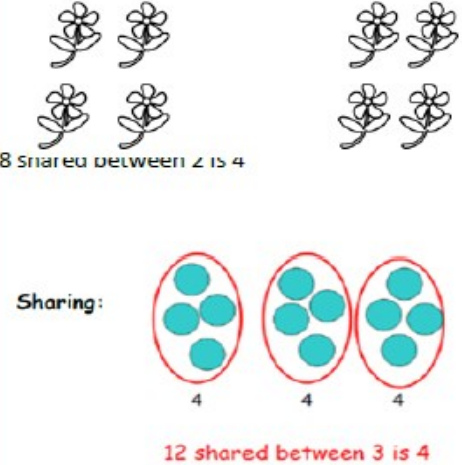
Year 5 - Year 6 - Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract																																																										
<p>Column Multiplication for 3 and 4 digits x 1 digit.</p>	<div style="display: flex; align-items: center;"> <table border="1" style="border-collapse: collapse; text-align: center; margin-right: 10px;"> <tr> <td style="background-color: red; color: white;">Hundreds</td> <td style="background-color: green; color: white;">Tens</td> <td style="background-color: blue; color: white;">Ones</td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> <tr> <td></td> <td></td> <td></td> </tr> </table> <div style="font-size: small;"> <p>It is important at this stage that they always multiply the ones first.</p> <p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p> </div> </div>	Hundreds	Tens	Ones													<table border="1" style="border-collapse: collapse; text-align: center; margin-bottom: 10px;"> <tr> <td>x</td> <td>300</td> <td>20</td> <td>7</td> </tr> <tr> <td>4</td> <td>1200</td> <td>80</td> <td>28</td> </tr> </table>	x	300	20	7	4	1200	80	28	<div style="text-align: right; margin-bottom: 10px;"> $\begin{array}{r} 327 \\ \times 4 \\ \hline 28 \\ 80 \\ 1200 \\ \hline 1308 \end{array}$ </div> <table border="1" style="border-collapse: collapse; text-align: center; margin-bottom: 10px;"> <tr><td></td><td>3</td><td>2</td><td>7</td></tr> <tr><td>x</td><td></td><td></td><td>4</td></tr> <tr><td></td><td>1</td><td>3</td><td>0</td></tr> <tr><td></td><td></td><td>1</td><td>2</td></tr> </table> <p style="font-size: small;">This will lead to a compact method.</p>		3	2	7	x			4		1	3	0			1	2																			
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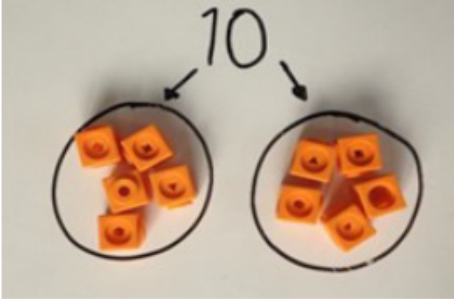

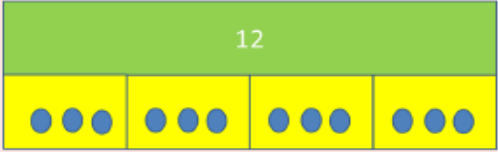


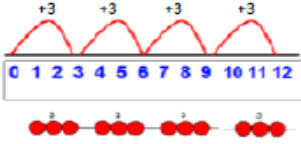
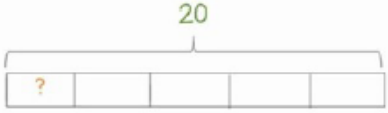
Year 6 - Multiplication

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.			<p>Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer.</p> $\begin{array}{r} 3.19 \\ \times 8 \\ \hline 25.52 \end{array}$

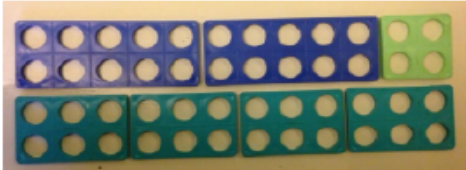


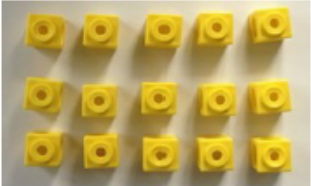
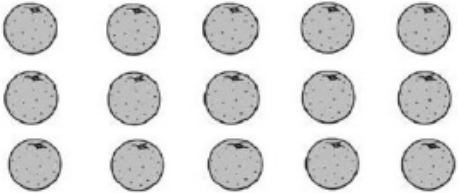
Year 1 - Division

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p> <p><i>Use Gordon ITPs for modelling</i></p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>8 shared between 2 is 4</p> <p>Sharing:</p> <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is 4</p> <p style="text-align: center;">4</p>

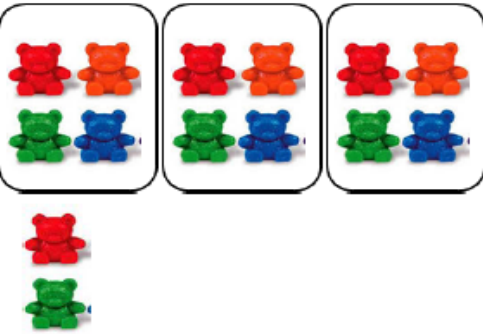


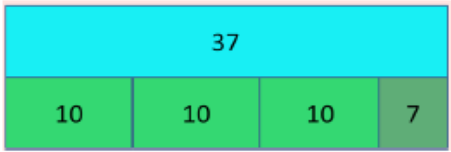
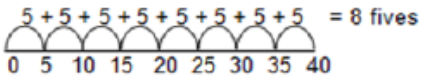
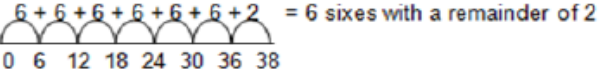
Year 2 - Division

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as sharing</p>	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p style="text-align: center;">$8 \div 2 = 4$</p> <p>Children use bar modelling to show and support understanding.</p>  <p style="text-align: center;">$12 \div 4 = 3$</p>	<p style="text-align: center;">$12 \div 3 = 4$</p>
<p>Division as grouping</p>	<p>Divide quantities into equal groups.</p> <p>Use cubes, counters, objects or place value counters to aid understanding.</p>  	<p>Use number lines for grouping</p>  <p style="text-align: center;">$12 \div 3 = 4$</p> <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  <p style="text-align: center;">$20 \div 5 = ?$ $5 \times ? = 20$</p>	<p>$28 \div 7 = 4$</p> <p>Divide 28 into 7 groups. How many are in each group?</p>

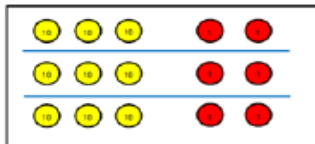
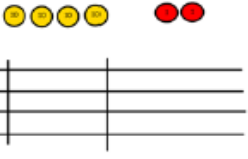

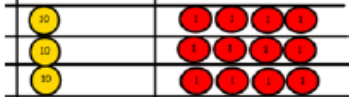
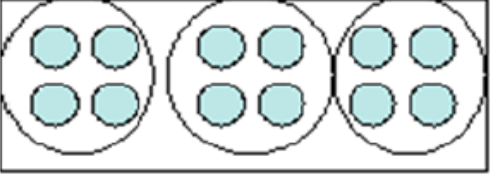
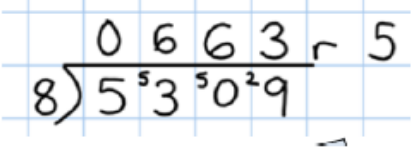
Year 3 - Division

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division as grouping</p>	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> $96 \div 3 = 32$ 	<p>Continue to use bar modelling to aid solving division problems.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	<p>How many groups of 6 in 24?</p> $24 \div 6 = 4$
<p>Division with arrays</p>	 <p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p>	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> $7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$

Year 3 - Division continued

Objective & Strategy	Concrete	Pictorial	Abstract
<p>Division with remainders.</p>	<p>$14 \div 3 =$</p> <p>Divide objects between groups and see how much is left over</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p> 	<p>Complete written divisions and show the remainder using r.</p> $29 \div 8 = 3 \text{ REMAINDER } 5$ <p style="text-align: center;"> ↑ ↑ ↑ ↑ dividend divisor quotient remainder </p>
		<p>Example without remainder: $40 \div 5$ Ask "How many 5s in 40?"</p>  <p>Example with remainder: $38 \div 6$</p> 	
		<p>For larger numbers, when it becomes inefficient to count in single multiples, bigger jumps can be recorded using known facts.</p>	

Year 4 - Year 6 - Division

Objective & Strategy	Concrete	Pictorial	Abstract				
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>Short Division</p>	<p>$96 \div 3$</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: center;">Tens</td> <td style="text-align: center;">Units</td> </tr> <tr> <td style="text-align: center;">3</td> <td style="text-align: center;">2</td> </tr> </table>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p style="text-align: right;">Calculations $42 \div 3$</p> <p>$42 \div 3 =$</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over.</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	Tens	Units	3	2	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder.</p> $\begin{array}{r} 218 \\ 3 \overline{) 872} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 432} \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 16 \overline{) 511.0} \end{array}$ 
Tens	Units						
3	2						

Year 6 - Division

Long Division

Step 1—a remainder in the ones

$$\begin{array}{r} \text{h t o} \\ 041 \text{ R}1 \\ \hline 4 \overline{) 165} \end{array}$$

4 does not go into 1 (hundred). So combine the 1 hundred with the 6 tens (160).

4 goes into 16 four times.

4 goes into 5 once, leaving a remainder of 1.

$$\begin{array}{r} \text{th h t o} \\ 0400 \text{ R}7 \\ \hline 8 \overline{) 3207} \end{array}$$

8 does not go into 3 of the thousands. So combine the 3 thousands with the 2 hundreds (3,200).

8 goes into 32 four times ($3,200 \div 8 = 400$)

8 goes into 0 zero times (tens).

8 goes into 7 zero times, and leaves a remainder of 7.

Year 6 - Division

Long Division

Step 1 continued...

$$\begin{array}{r} \text{h t o} \\ 061 \\ 4 \overline{) 247} \\ \underline{-4} \\ 3 \end{array}$$

When dividing the ones, 4 goes into 7 one time. Multiply $1 \times 4 = 4$, write that four under the 7, and subtract. This finds us the remainder of 3.

Check: $4 \times 61 + 3 = 247$

$$\begin{array}{r} \text{th h t o} \\ 0402 \\ 4 \overline{) 1609} \\ \underline{-8} \\ 1 \end{array}$$

When dividing the ones, 4 goes into 9 two times. Multiply $2 \times 4 = 8$, write that eight under the 9, and subtract. This finds us the remainder of 1.

Check: $4 \times 402 + 1 = 1,609$

Year 6 - Division

Long Division

Step 2—a remainder in the tens

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{4} \\ 18 \end{array}$ <p>Two goes into 5 two times, or 5 tens $\div 2 = 2$ whole tens -- but there is a remainder!</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>To find it, multiply $2 \times 2 = 4$, write that 4 under the five, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the leftover 1 ten. You combine the remainder ten with 8 ones, and get 18.</p>

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract.</p>	$\begin{array}{r} \text{t o} \\ 2 \overline{) 58} \\ \underline{-4} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>The division is over since there are no more digits in the dividend. The quotient is 29.</p>

Year 6 - Division

Long Division

Step 2—a remainder in any of the place values

1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ \underline{2} \\ 0 \end{array}$ <p>Two goes into 2 one time, or 2 hundreds $\div 2 = 1$ hundred.</p>	$\begin{array}{r} \text{h t o} \\ 1 \\ 2 \overline{) 278} \\ \underline{-2} \\ 0 \end{array}$ <p>Multiply $1 \times 2 = 2$, write that 2 under the two, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 18 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \end{array}$ <p>Next, drop down the 7 of the tens next to the zero.</p>
Divide.	Multiply & subtract.	Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \end{array}$ <p>Divide 2 into 7. Place 3 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 1 \end{array}$ <p>Multiply $3 \times 2 = 6$, write that 6 under the 7, and subtract to find the remainder of 1 ten.</p>	$\begin{array}{r} \text{h t o} \\ 13 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Next, drop down the 8 of the ones next to the 1 leftover ten.</p>
1. Divide.	2. Multiply & subtract.	3. Drop down the next digit.
$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \end{array}$ <p>Divide 2 into 18. Place 9 into the quotient.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>Multiply $9 \times 2 = 18$, write that 18 under the 18, and subtract to find the remainder of zero.</p>	$\begin{array}{r} \text{h t o} \\ 139 \\ 2 \overline{) 278} \\ \underline{-2} \\ 07 \\ \underline{-6} \\ 18 \\ \underline{-18} \\ 0 \end{array}$ <p>There are no more digits to drop down. The quotient is 139.</p>

Maths vocabulary glossary - Addition and Subtraction

Addend - A number to be added to another.

Aggregation - combining two or more quantities or measures to find a total.

Augmentation - increasing a quantity or measure by another quantity.

Commutative - numbers can be added in any order.

Complement - in addition, a number and its complement make a total e.g. 300 is the complement to 700 to make 1,000

Difference - the numerical difference between two numbers is found by comparing the quantity in each group.

Exchange - Change a number or expression for another of an equal value.

Minuend - A quantity or number from which another is subtracted.

Partitioning - Splitting a number into its component parts.

Reduction - Subtraction as take away.

Subitise - Instantly recognise the number of objects in a small group without needing to count.

Subtrahend - A number to be subtracted from another.

Sum - The result of an addition.

Total - The aggregate or the sum found by addition.

Maths vocabulary glossary - Multiplication and Division

Array – An ordered collection of counters, cubes or other item in rows and columns.

Commutative – Numbers can be multiplied in any order.

Dividend – In division, the number that is divided.

Divisor – In division, the number by which another is divided.

Exchange – Change a number or expression for another of an equal value.

Factor – A number that multiplies with another to make a product.

Multiplicand – In multiplication, a number to be multiplied by another.

Partitioning – Splitting a number into its component parts.

Product – The result of multiplying one number by another.

Quotient – The result of a division

Remainder – The amount left over after a division when the divisor is not a factor of the dividend.

Scaling – Enlarging or reducing a number by a given amount, called the scale factor