# Autumn Scheme of Learning



# #MathsEveryoneCan

2020-21





## New for 2020/21

2020 will go down in history. The world has changed for all of us.

We want to do as much as we can to support children, teachers, parents and carers in these very uncertain times.

We have amended our schemes for 2020/21 to:

- $\star$  highlight key teaching points
- ★ recap essential content that children may have forgotten
- ★ flag any content that you might not have covered during the school closures period.

We hope these changes will add further value to the schemes and save you time.



## Lesson-by-lesson overviews

We've always been reluctant to produce lesson-bylesson overviews as every class is individual and has different needs. However, many of you have said that if blended learning becomes a key feature of school life next year, a weekly plan with linked content and videos could be really useful.

As always, we've listened! We've now produced a complete lesson-by-lesson overview for Y1 to Y9 that schools can use or adapt as they choose. Each lesson will be linked to a free-to-use home learning video, and for premium subscribers, a worksheet. This means that you can easily assign work to your class, whether they are working at home or in school.

Inevitably, this lesson-by-lesson structure won't suit everyone, but if it works for you, then please do make use of this resource as much as you wish.

#### White Rose Maths

# **Teaching for Mastery**

These overviews are designed to support a mastery approach to teaching and learning and have been designed to support the aims and objectives of the new National Curriculum.

The overviews:

- have number at their heart. A large proportion of time is spent reinforcing number to build competency
- ensure teachers stay in the required key stage and support the ideal of depth before breadth
- ensure students have the opportunity to stay together as they work through the schemes as a whole group
- provide plenty of opportunities to build reasoning and problem solving elements into the curriculum.

For more guidance on teaching for mastery, visit the NCETM website:

https://www.ncetm.org.uk/resources/47230

# Concrete - Pictorial - Abstract

We believe that all children, when introduced to a new concept, should have the opportunity to build competency by taking this approach.

**Concrete** – children should have the opportunity to use concrete objects and manipulatives to help them understand what they are doing.

Pictorial – alongside this children should use pictorial representations. These representations can then be used to help reason and solve problems.

Abstract – both concrete and pictorial representations should support children's understanding of abstract methods.

Need some CPD to develop this approach? Visit <u>www.whiterosemaths.com</u> for find a course right for you.

## **Supporting resources**

#### NEW for 2019-20!

We have produced supporting resources for every small step from Year 1 to Year 11.

The worksheets are provided in three different formats:

- Write on worksheet ideal for children to use the ready made models, images and stem sentences.
- Display version great for schools who want to cut down on photocopying.
- PowerPoint version one question per slide. Perfect for whole class teaching or mixing questions to make your own bespoke lesson.

For more information visit our online training and resources centre <u>resources.whiterosemaths.com</u> or email us directly at <u>support@whiterosemaths.com</u>





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## **Meet the Characters**

Children love to learn with characters and our team within the scheme will be sure to get them talking and reasoning about mathematical concepts and ideas. Who's your favourite?





	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week	9 Week 10	Week 11	Week 12
Autumn	Numb	er: Place	Value	Nur	nber: Ad	dition and	J Subtrac	tion	Number: Multiplication Division		and	
Spring	Numbe ai	er: Multipl nd Divisio	lication on	Measurement: Money	Sta	atistics	Mea Lei Pé	suremen ngth and erimeter	t:	Number: Fi	ractions	Consolidation
Summer	Num	ber: Frac	tions	Measi	urement:	Time	Geon Propei Sha	netry: rties of ape	Measurement: Mass and Capacity		Consolidation	



#### Year 3 | Autumn Term | Week 1 to 3 – Number: Place Value



# Overview Small Steps

Represent numbers to 100	R
Tens and ones using addition	R
Hundreds	
Represent numbers to 1,000	
100s, 10s and 1s (1)	
100s, 10s and 1s (2)	
Number line to 1,000	
Find 1, 10, 100 more or less than a given number	
Compare objects to 1,000	
Compare numbers to 1,000	
Order numbers	
Count in 50s	

#### Notes for 2020/21

Children should already have some understanding of tens and ones from Y2, however it may be useful to recap this content before exploring hundreds.

You may want to ensure that you use plenty of examples of numbers within 100 including number lines to 100 before moving on to the number line to 1,000



#### **Represent Numbers to 100**

#### **Notes and Guidance**

Children need to be able to represent numbers to 100 using a range of concrete materials, such as bead strings, straws, Base 10 equipment etc.

Children should also be able to state how a number is made up. For example, they can express 42 as 4 tens and 2 ones or as 42 ones.

Mathematical Talk

How have the beads been grouped? How does this help you count?

Can you show me the tens/ones in the number?

Which resource do you prefer to use for larger numbers? Which is quickest? Which would take a long time?

#### Varied Fluency

Here is part of a bead string.



Complete the sentences.

There are tens and ones.

The number is \_\_\_\_\_.

Represent 45 on a bead string and complete the same sentence stems.





One ten and five ones Thirty-five



Represent 67 in three different ways.



#### **Represent Numbers to 100**

#### **Reasoning and Problem Solving**



#### 70, 20, 72, 27

The largest number is 72

The smallest number is 20

Because it would make a 1 digit number.



#### Tens and Ones (2)

#### Notes and Guidance

Children continue to use a part-whole model to explore how tens and ones can be partitioned and recombined to make a total.

Children will see numbers partitioned in different ways. For example, 39 written as 20 + 19

This small step will focus on using the addition symbol to express numbers to 100. For example, 73 can be written as 70 + 3 = 73

#### Mathematical Talk

What clues are there in the calculations? Can we look at the tens number or the ones number to help us?

What number completes the part-whole model?

What is the same/different about the calculations?

What are the key bits of information? Can you draw a diagram to help you?

#### Varied Fluency



10 + 420 + 19

80 + 1





40 + 0

39

Complete the part-whole model and write four number sentences to match.



- Dora has 20 sweets and Amir has 15 sweets. Represent the total number of sweets:
  - With concrete resources.
  - In a part-whole model.
  - As a number sentence.



#### Tens and Ones (2)

#### Reasoning and Problem Solving

#### Teddy thinks that,



Explain the mistake he has made.

Can you show the correct answer using concrete resources?

40 + 2 = 42Teddy has just combined the numbers to make 402 without thinking about their place value.

Fill in the missing numbers.	1  ten + 3  ones = 13 2 tens + 3 ones =
1 ten + 3 ones = 13	23 3 tens + 3 ones =
2 tens + ones = 23	4 tens + 3 ones = 43
3 tens + 3 ones =	
tens + 3 ones = 43	
What would the next number in the pattern be?	5 tens + 3 ones = 53



#### Hundreds

#### Notes and Guidance

- Children build on their understanding of tens and link this to 100
- This is the first time they explore 100 explicitly. It is crucial children understand that ten tens make 100 and a hundred ones make 100
- They use a variety of concrete equipment to see this relationship. Once children understand the concept of 100, they will count objects and numbers in multiples of 100 up to 1,000

### Mathematical Talk

- How many tens have you made? How else can we say this?
- What do these digits represent?
- How many ones have you made? How else can you say this?
- If we continue counting in tens, what do we say after 100?
- What numbers wouldn't we say?

## Varied Fluency

- Use bundles of straws in tens, bead strings and Base 10 to explore how many tens make a hundred. Children use the equipment to count up and down in tens to make 100
- There are <u>3 tens</u> this is <u>thirty</u>. There are \_\_\_\_\_ this is \_\_\_\_\_ . There are \_\_\_\_\_ tens in one hundred.
- There are 100 sweets in each jar.



- How many sweets are there altogether? Write your answer in numerals and words.
- Complete the number tracks.





#### Hundreds

#### Reasoning and Problem Solving

True or False?	True, because if you start with zero	Whitney thinks the place v showing the number eight.	Whitney is incorrect becau	
If I count in 100s from zero, all of the numbers will be even. Convince me.	and add 100 you get an even number, and you are adding another even so the number will always be even.	HundredsTensImage: Constraint of the second	Ones	there are eight counters in the hundreds colur so they represe eight hundreds. The number is 800
<ul> <li>Sort these statements into always, sometimes or never.</li> <li>When counting in hundreds, the ones column changes.</li> <li>When counting in hundreds, the hundreds column changes.</li> <li>To count in hundreds we use 3-digit numbers.</li> </ul>	<ul><li>Never</li><li>Always</li><li>Sometimes</li></ul>	Do you agree? Explain why Using all of the counters, w smallest number you can r What other numbers could	/. /hat is the make? I you make?	The smallest number that ca be made is 8 Other possible numbers includ 80 170 350 etc.

14



#### Numbers to 1,000

#### Notes and Guidance

In this small step, children will primarily use Base 10 to become familiar with any number up to 1,000

Using Base 10 will emphasise to children that hundreds are bigger than tens and tens are bigger than ones.

Children need to see numbers with zeros in different columns, and show them with concrete and pictorial representations.

#### Mathematical Talk

Does it matter which order you build the number in?

Can you have more than 9 of the same type of number e.g. 11 tens?

Can you create a part-whole model using or drawing Base 10 in each circle?

#### Varied Fluency

15

Write down the number represented with Base 10 in each case.



Mo is drawing numbers. Can you complete them for him?





#### Numbers to 1,000

#### Reasoning and Problem Solving

Teddy has used Base 10 to represent the number 420. He has covered some of them up.



Work out the amount he has covered up.

How many different ways can you make the missing amount using Base 10?

110	is	the	missing
amo	U	nt.	

Possible ways:

- 1 hundred and 1 ten
- 11 tens
- 110 ones
- 10 tens and 10 ones
- 50 ones and 6 tens etc.



Explain how you know.

Dora and Mo have both made the number 315, but represented it differently.

3 hundreds, 1 ten and 5 ones is the same as 2 hundreds, 10 tens and 15 ones.



## 100s, 10s and 1s (1)

#### Notes and Guidance

Children should understand that a 3-digit number is made up of 100s, 10s and 1s.

They read numbers shown in different representations on a place value grid, and write them in numerals.

They should be able to represent different 3-digit numbers in various ways such as Base 10 or numerals.

#### Mathematical Talk

What is the value of the number shown on the place value chart?

Why is it important to put the values into the correct column on the place value chart?

How many more are needed to complete the place value chart?

Can you make your own numbers using Base 10? Ask a friend to tell you what number you have made.

## Varied Fluency

What is the value of the number represented in the place value chart?

Hundreds	Tens	Ones

Write your answer in numerals and in words.

Complete this	place value chart s	o that it shows t	he number 354
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Hundreds	Tens	Ones

Represent the number using a part-whole model.

How many different ways can you make the number 452? Can you write each way in expanded form? (e.g. 400 + 50 + 2)

Compare your answer with a partner.



## 100s, 10s and 1s (1)

#### Reasoning and Problem Solving

Hundreds	Tens	Ones	Possible answers:
			I disagree because there are six hundreds, four tens and seven ones so the number is 647.
Eva	I notice that 647 and 467 have the same digits but in a different order so the digits have different values.		
Is Eva correct? What do you n shown?			

# wers: ecause



3

Using each digit card, which numbers can you make?

Use the place value grid to help.

Hundreds	Tens	Ones

Compare your answers with a partner.

# The numbers that can be made are:

- 503
- 530
- 305
- 350
- (0)35
- (0)53



## 100s, 10s and 1s (2)

#### Notes and Guidance

Children use place value counters to represent different numbers and understand how a number is made.

Their work with Base 10 should help them understand that the hundreds counter is worth more than the tens counter and the tens counter is worth more than the ones counter.

Mathematical Talk

- What is the same and what is different about Base 10 and place value counters?
- Why do we not call this number 300506?
- What number would be shown if 1/10/100 was added?
- Why is it important to put the values into the correct column on the place value grid?
- What do we need to do if there is a zero in the number we are representing?

## Varied Fluency

What number is shown on the place value chart?



If one more 10 is added, what number would be shown?

Use place value counters and a place value grid to represent the numbers:







## 100s, 10s and 1s (2)

#### **Reasoning and Problem Solving**





#### Number Line to 1,000

#### **Notes and Guidance**

Children estimate, work out and write numbers on a number line.

Number lines should be shown with or without start and end numbers, and with numbers already placed on it.

Children may still need Base 10 and/or place values to work with as they develop their understanding of the number line.

#### Mathematical Talk

What is the value of each interval on the number line?

Which side of the number line did you start from? Why?

When estimating where a number should be placed, what facts can help you?

Can you draw a number line where 600 is the starting

number, and 650 is half way along?

What do you know about the number that A is representing? A is more/less than

What value can A definitely not be? How do you know?

#### Varied Fluency





#### Number Line to 1,000

#### Reasoning and Problem Solving

Estimate where seven hundred and twenty-five will go on each of the number lines.



Explain why it is not in the same place on each number line.

725 is in different places because each line has different numbers at the start and end so the position of 725 changes.

All three of the number lines have different scales and therefore the difference between 725 and the starting and finishing number is different on all three number lines.





## 1, 10, 100 More or Less

#### Notes and Guidance

Building on children's learning in Year 2 where they explored finding one more/less, children now move onto finding 10 and 100 more or less than a given number.

Show children that they can represent their answer in a variety of different ways. For example, as numerals or words, or with concrete manipulatives.

#### Mathematical Talk

- What is 10 more than/less than \_\_\_\_?
- What is 100 more than/less than \_\_\_\_?

Which column changes? Can more than one column change?

What happens when I subtract 10 from 209? Why is this more difficult?

## Varied Fluency





Show ten more and ten less than the following numbers using Base 10 and place value counters.

550	724	302

#### Complete the table.

	100 less	Number	100 more
		100	
23			©White Rose Maths



### 1, 10, 100 More or Less

#### Reasoning and Problem Solving

10 more than my number is the same as 100 less than 320	The number described is 210		A counter is m chart.	Possible answers: 401		
What is my number?	than 320 is 220,		Hundreds	Tens	Ones	302
Explain how you know.	which means 220 is 10 more than the original					
Write your own similar problem to describe the original number.	number.					
l think of a number, add ten, subtract one hundred and then add one.	The start number was 345 because one less than 256		What number	could it have t	been?	
My answer is 256	is 255, one					
What number did I start with?	than 255 is 355					
Explain how you know.	355 is 345					
What can you do to check?	To check I can follow the steps back to get 256					



#### **Compare Objects**

#### Notes and Guidance

Children use objects to represent numbers to 1,000 When given two numbers represented by objects, they use comparative language and symbols to determine which is greatest/smallest. Children can make the numbers using concrete manipulatives and draw them pictorially. Use stem sentences to ensure the correct vocabulary is being used e.g. \_\_\_\_\_ is greater than \_\_\_\_\_.

Mathematical Talk

How do you know which number is greater? Do you start counting hundreds, tens or ones first? Why?

What strategy did you use to compare the two numbers? Is this the same or different to your partner?

Are the Base 10 and place value counters showing the same amount? How do you know?

Is there only one answer?

#### Varied Fluency





#### **Compare Objects**

#### Reasoning and Problem Solving

#### Which image is the odd one out?



539 540	541	542	543	544
---------	-----	-----	-----	-----





Explain why. How else can you represent the number? The part-whole model is the odd one out because it shows 643 whereas all the other images show 543

Children could show 543 in a part-whole model correctly, in Base 10 a different way or with place value counters in a different way.

#### True or False?



Explain your answer.

The image is not correct because the number 244 is represented on both sides of the inequality symbol.

An equal sign should have been used.

The number on the left must be made larger or the number on the right must be made smaller, to make this true.



#### Compare Numbers

#### Notes and Guidance

Children compare numbers presented as numerals rather than objects.

They need to be encouraged to use previous learning to choose an efficient method to compare the numbers. For example, children may choose to place the numbers on a number line, make them using concrete manipulatives or draw them in a place value chart to compare.

#### Mathematical Talk

What strategy did you use to compare the numbers?

What materials would be useful to help you compare the numbers?

How do you know which number is the smallest /greatest? Which column do you start comparing from? Why?

Can you find more than one way to complete the statements?

#### Varied Fluency





#### **Compare Numbers**

#### Reasoning and Problem Solving

#### Amir has 3 jars of sweets.



Jar A contains 235 sweets.

Jar C contains 175 sweets.



Jar B could contain any number of sweets between 176 and 234 inclusive.

Discussion point: Could B contain 175 or 235 sweets? Why?

I am thinking of a number.	446 or 464
It is between 300 and 500	The only
The digits add up to 14	possibilities to go in the hundreds column are 3 and
The difference between the greatest digit	4
and the smallest digit is 2	lf it was 3, the
What could my number be?	other two digits would have to total
Is there only one option?	these pairs give
Explain each step of your working.	difference between the greatest and smallest digit, so the number has to have 4 in the hundreds column.



#### **Order Numbers**

#### Notes and Guidance

Children explore ordering a set of numbers from smallest to greatest and greatest to smallest. They need to be able to explain their reasoning throughout. They could still use Base 10 or other concrete materials to help them to make decisions about ordering.

At this point, children are introduced to the words ascending and descending.

#### Mathematical Talk

How do you know you have created the greatest/smallest number?

What number is being represented by the place value counters/Base 10?

What does the word ascending/descending mean?

Can you find more than one way to order your numbers?

## Varied Fluency

Here are three digit cards.



What is the greatest number you can make? What is the smallest number you can make?





Here is a list of numbers.

312, 321, 123, 132, 213, 231

Place the numbers in ascending order. Now place them in descending order. What do you notice?



#### **Order Numbers**

#### Reasoning and Problem Solving

Whitney has six different numbers.

She put them in ascending order then accidentally spilt some ink onto her page. Two of her numbers are now covered in ink.

214, 243,	256, 💥 289
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What could the hidden numbers be? Explain how you know. The first number could be anything between 215 and 242

The second hidden number could be anywhere between 257 and 288

#### True or False?

When ordering numbers you only need to look at the place value column with the highest value. False.

For example, if you are ordering numbers in the hundreds you should start by looking at the hundreds column, but sometimes two numbers will have the same number of hundreds and so you will also need to look at other columns.



# Count in 50s Notes and Guidance Varied Fluency Children use their knowledge of the patterns in the 5 times table to count in steps of 50 Look at the number patterns. What do you notice? They should start from any given multiple of 50 and be able to count both forwards and backwards. 5 10 15 20 25 30

## Mathematical Talk

What is the same and what is different between counting in 5s and counting in 50s?

Hence, what is the connection between the 5 times table and the 50 times table?

Can you notice a pattern as the numbers increase/decrease?

Can you correct the mistakes in each?

50	100	150	200	250	300
----	-----	-----	-----	-----	-----



50		150	200			350		450	
----	--	-----	-----	--	--	-----	--	-----	--

750	700	650		500		350



50, 100, 105, 200, 250, 300 ...

990, 950, 900, 850, 800 ...



#### Count in 50s

#### Reasoning and Problem Solving

Odd One Out 100, 150, 200, 215, 300 Circle the odd one out. Explain how you know.	215 is the odd one out because it is not a multiple of 50 If we were counting up in 50s from 100, it should have been 250 not 215	<ul> <li>Always, Sometimes, Never</li> <li>Sort the statements into always, sometimes or never.</li> <li>When counting in 50s starting from 0, the numbers are all even.</li> <li>There are only two digits in a</li> </ul>	<ul><li>Always</li><li>Sometimes</li></ul>
Which is quicker: counting to 50 in 10s or counting to 150 in 50s? Explain your answer.	It is quicker to count to 150 in 50s as it would only be 3 steps whereas counting to 50 in 10s would be 5 steps.	<ul> <li>Multiple of 50</li> <li>Only the hundreds and tens column changes when counting in 50s.</li> </ul>	• Sometimes



#### Year 3 | Autumn Term | Week 4 to 8 – Number: Addition & Subtraction



# Overview Small Steps

#### Add and subtract multiples of 100 Add and subtract 1s R Add and subtract 3-digit and 1-digit numbers - not crossing 10 Add a 2-digit and 1-digit number - crossing 10 R Add 3-digit and 1-digit numbers - crossing 10 R Subtract a 1-digit number from 2-digits - crossing 10 Subtract a 1-digit number from a 3-digit number - crossing 10 Add and subtract 3-digit and 2-digit numbers - not crossing 100 Add 3-digit and 2-digit numbers - crossing 100 Subtract a 2-digit number from a 3-digit number - crossing 100 Add and subtract 100s Spot the pattern - making it explicit R Add two 2-digit numbers - crossing 10 - add ones & add tens Subtract a 2-digit number from a 2-digit number - crossing 10

## Notes for 2020/21

Children should have met addition and subtraction of 2digits + 2-digits, although it may not be embedded and they may not have met the formal column method.

We have added steps that provide opportunity for recap/introduce the formal method of 2-digits + 2-digits.



#### Year 3 | Autumn Term | Week 4 to 8 – Number: Addition & Subtraction

# Overview Small Steps

#### Add and subtract a 2-digit and 3-digit numbers – not crossing 10 or 100

- Add a 2-digit and 3-digit numbers crossing 10 or 100
- Subtract a 2-digit number from a 3-digit number crossing 10 or 100
- Add two 3-digit numbers not crossing 10 or 100
- Add two 3-digit numbers crossing 10 or 100
- Subtract a 3-digit number from a 3-digit number no exchange
- Subtract a 3-digit number from a 3-digit number exchange
- Estimate answers to calculations
  - Check answers

#### Notes for 2020/21

Use the early steps in this unit to recap place value of 2-digit and 3-digit numbers.

You may want to omit the estimate and check answers steps and instead embed this throughout the other steps.





#### Add & Subtract Multiples of 100

#### Notes and Guidance

Children are introduced to adding numbers greater than 100

They will apply their prior knowledge of adding and subtracting ones and tens to adding and subtracting multiples of 100

Using concrete manipulatives and pictorial representations throughout is important so the children can see the value of the digits.

#### Mathematical Talk

What is the same and what is different about 2 ones and 3 ones, 2 tens and 3 tens and 2 hundreds and 3 hundreds?

What is \_\_\_\_\_ hundreds and \_\_\_\_\_ hundreds equal to?

How many different ways can you represent 200 + 300?

#### Varied Fluency

Complete:



2 ones and 3 ones is equal to \_\_\_\_ ones.



2 tens and 3 tens is equal to \_\_\_\_\_ tens.

2 hundi

2 hundreds and 3 hundreds is equal to \_\_\_\_\_ hundreds.

#### Complete each box for 400 + 500








# Add & Subtract Multiples of 100

# Reasoning and Problem Solving

+ = 800	0 + 800
Each of the missing numbers are multiples of 100 Find all the possible missing numbers.	100 + 700 $200 + 600$ $300 + 500$ $400 + 400$ $500 + 300$ $600 + 200$ $700 + 100$
161  know  700 - 500 - 200  what else	800 + 0 Children may write
do I know? Show me using concrete and pictorial representations.	all the related facts and link it to a bar model. They may also



answer is not 800



## Add and Subtract 1s

#### Notes and Guidance

Children should start seeing the pattern when we add and subtract 1 and comment upon what happens.

This is the step before finding ten more than or ten less than, as bridging beyond a 10 should not be attempted yet.

The pattern should be highlighted also by adding 2 (by adding another one) and then adding 3

### Mathematical Talk

- What happens when we add 2?
- What is the link between adding 1 and adding 2?
- What about if we want to add 3?
- How can a bead string help when we are adding 1, 2, 3 etc.?
- Where will be the best place to start on each number track? Why?

### Varied Fluency

Create sentences based on the picture.



#### Example

There are 4 children playing in a park. One more child joins them so there will be 5 children playing together.

Continue the pattern

22 = 29 - 722 = 28 - 6

Can you create an addition pattern by adding in ones and starting at the number 13?

#### Continue the number tracks below.







13		



## Add and Subtract 1s

## Reasoning and Problem Solving

True or False?			Jack's house				
These four calculatic answer.	ons have the same	True, because they all equal 7 and	Annie's house				
1+4+2	4 + 2 + 1	addition is commutative.					
2 + 4 + 1	4 + 1 + 2		Jack lives 5 km from school. Annie lives 4 km from school in the same direction.				
These four calculatic answer.	ons have the same	False, because	False, because	False, because	False, because	What is the distance between Jack and Annie's houses?	1 km No. he will walk 2
7 – 3 – 2	2 – 3 – 7	subtraction isn't commutative.	After travelling to and from school, Jack thinks that he will walk 1 km more than	km further. 1 km			
3 – 2 – 7	7 – 2 – 3		Annie. Is he correct? Explain your answer.	school and 1 km on the way home.			
			What will be the difference in distance walked after 2 school days?	4 km			



# 3-digit & 1-digit Numbers

#### Notes and Guidance

During this small step, children add and subtract ones from a 3-digit number without an exchange. They consider which digits are affected when adding ones. For example, if a child is completing 214 - 3 and 214 + 3 they see that they just need to focus on the ones column. Therefore, all they need to do is 4 + 3 and 4 - 3 respectively.

The use of the column method can be used but mental arithmetic is the best strategy.

### Mathematical Talk

Which column do I need to focus on?

What is the same about the subtractions? What changes each time? Write the number sentence that would come next in each list. Can you write the number sentence that would come before?

Can you use < and > to compare Jack and Tommy's team points?

## Varied Fluency

Hun	dreds	Tens	Ones

Use the place value grid to complete the calculations.

214 - 3 = \_\_\_\_ 214 + 3 = \_\_\_\_

Complete:

356 - 5 =	356 - 5 =	356 - 5 =
357 - 5 =	356 - 4 =	366 - 5 =
358 - 5 =	356 – 3 =	376 – 5 =
359 - 5 =	356 – 2 =	386 - 5 =

Jack has 534 team points and gets four more. Tommy has 534 team points and loses four of his. How many team points does each person have? Who has the most?



# **3-digit & 1-digit Numbers**





# Add 2-digits and 1-digit

#### Notes and Guidance

Before crossing the 10 with addition, children need to have a strong understanding of place value. The idea that ten ones are the same as one ten is essential here. They need to be able to count to 20 and need to be able to partition two-digit numbers in order to add them. They need to understand the difference between one-digit and two-digit numbers and line them up in columns. In order to progress to using the number line more efficiently, children need to be secure in their number bonds.

### Mathematical Talk

Using Base 10, can you partition your numbers?

Can we exchange 10 ones for one ten?

How many ones do we have? How many tens do we have?

Can you draw the Base 10 and show the addition pictorially?

### Varied Fluency

#### 17 + 5 =



Can you put the larger number in your head and count on the smaller number? Start at 17 and count on 5

Can we use number bonds to solve the addition more efficiently?



We can partition 5 into 3 and 2 and use this to bridge the 10



Find the total of 28 and 7



- Partition both the numbers.
- Add together the ones.
- Have we got 10 ones?
- Exchange 10 ones for 1 ten.
- How many ones do we have?
- How many tens do we have?



## Add 2-digits and 1-digit

#### **Reasoning and Problem Solving**

#### Always, Sometimes, Never

I am thinking of a twodigit number, if I add ones to it, I will only need to change the ones digit.

Explain your answer.

Sometimes, because if your ones total 10 or more you will have to exchange them which will change the tens digit.

#### Here are three digit cards.



Place the digit cards in the number sentence.

How many different totals can you find?



What is the smallest total?

What is the largest total?

67 + 8 = 75 68 + 7 = 75 76 + 8 = 84 78 + 6 = 84 86 + 7 = 9387 + 6 = 93

75 is the smallest total.

93 is the largest total.





# Add 3-digit & 1-digit Numbers

### Notes and Guidance

Children add ones to a 3-digit number, with an exchange. They discover that when adding ones it can affect the ones column and the tens column

Children learn that we can only hold single digits in each column, anything over must be exchanged.

The use of 0 e.g. 145 - 5 is important so they know to use zero as a place holder.

## Mathematical Talk

When you add ones to a number does it always, sometimes or never affect the tens column?

What is the largest digit you can have in each column? Why?

How does using the number line support partitioning the number? What number bonds help us with this method?

## Varied Fluency



Use this method to calculate:

44



## Add 3-digit & 1-digit Numbers

Always, Sometimes, Never	Always	Which questions are harder to calculate?	The second and third are harder as
When 7 and 5 are added together in the	1+1	234 + 3 =	an exchange
ones column, the digit in the ones column of the answer will always be 2	2+0	506 + 8 =	needs to be made.
	9+5	455 + 7 =	
What other digits would always give a 2 in the ones column? Prove it.	0 <del>+</del> 4 6 <del>+</del> 6	521 + 6 =	
	will also always	Explain your answer.	
	give a 2 in the		
	ones colornin.		



# Subtract 1-digit from 2-digits

### Notes and Guidance

Just as with addition, children need to have a strong understanding of place value for subtraction. Children need to be able to count to 20 and need to be able to partition two-digit numbers in order to subtract from them. They need to understand the difference between one-digit and two-digit numbers and line them up in columns. In order to progress to using the number line more efficiently, children need to be secure in their number bonds.

## Mathematical Talk

Are we counting backwards or forwards on the number line?

Have we got enough ones to subtract?

Can we exchange a ten for ten ones?

How can we show the takeaway? Can we cross out the cubes?

#### Varied Fluency

#### 22 – 7 =



Can you put the larger number in your head and count back the smaller number? Start at 22 and count back 7

Can we use number bonds to subtract more efficiently?



We can partition 7 into 5 and 2 and use this to bridge the 10



#### Subtract 8 from 24



- Do we have enough ones to take 8 ones away?
  - Exchange one ten for ten ones. Take away 8 ones.
  - Can you write this using the column method?



R

# Subtract 1-digit from 2-digits

# **Reasoning and Problem Solving**

Jack and Eva are solving the subtraction Eva's method is Mo is counting back to solve 35 - 7Mo is not correct 23 - 9most efficient as he has included He counts because there are 35 when counting Here are their methods: less steps to take. back. 35, 34, 33, 32, 31, 30, 29 Jack The numbers are I put 9 in my head This is a common quite far apart so Is Mo correct? and counted on to 23  $\bigcirc$ mistake and can Jack's method of be modelled on a Explain your answer. finding the number line. difference takes a I put 23 in my head Match the number sentences to the long time and has and counted back 9 number bonds that make the method 42 - 5 -42 – 2 – 3 more room for more efficient. 43 - 3 - 3Eva 42 – 7 error. 42 - 2 - 343 - 3 - 5 42 - 543 - 8 -Who's method is the most efficient? 43 - 6 42 - 2 - 542 - 743 - 3 - 3Can you explain why? 43 - 843 - 3 - 5Can you think of another method to solve the subtraction. 43 - 642 - 2 - 5



#### Subtract 1-digit from 3-digits Varied Fluency Notes and Guidance Teddy uses Base 10 to calculate 321 - 4Children subtract a 1-digit number from a 3-digit number using an exchange. Children need to be secure in the fact that 321 is 3 hundreds, 2 tens and 1 one but that it is also 3 hundreds, 1 ten and 11 Use this method to calculate: 322 - 4322 - 7435 - 7If children are not secure with regrouping, it is important to revisit this before subtracting. Dora uses the part-whole model and number line to solve 132 - 4Mathematical Talk 125 126 127 128 129 130 131 132 133 134 How many ones do we exchange for one ten? 2 Why do all these subtractions require an exchange? When do Use this method to calculate: we not need to exchange? 132 - 8 123 – 8 123 - 5

Which method do you prefer? Can you calculate the subtractions mentally?

ones.

#### Red team have 672 points. Blue team have 7 fewer points than red team. How many points do blue team have?

#### 48



# Subtract 1-digit from 3-digits

### **Reasoning and Problem Solving**

Ron and Jack use Base 10 to solve 225 – 8

Ron's method:

Jack's method:



get the answer of 217 but I would choose Jack's because he has already exchanged one of his tens for ten ones.

Both methods can

Explain which method you would use and why.

Whitney might Whitney has 125 stickers. She gives less than 10 stickers to Eva. have given Eva 2, She has an odd number of stickers left. 4, 6 or 8 stickers. How many stickers might Whitney have All the answers given away? are even. What do you notice is the same about If Whitney had an your answers? even number of If Whitney had an even number of stickers left she stickers left, how many might she have might have given 1, given away? 3, 5, 7 or 9 away. Children explain Explain how you would solve these calculations: their methods, they may count on or back, use a 564 - = 558number line, partwhole model or - 8 = 725Base 10

352 = 361 -



# 3-digit & 2-digit Numbers

#### Notes and Guidance

Children look at what happens to a 3-digit number when a multiple of 10 is added or subtracted.

Different representations such as Base 10, arrow cards, place value charts should be used.

The use of the column method is exemplified in this example, but children should explore whether or not this is needed and explain why. Mental methods should be encouraged throughout.

### Mathematical Talk

How many tens can we add to 352 without exchanging? How many tens can we subtract from 352 without exchanging?

What patterns can you see between the additions and subtractions?

Can you see links between the columns?

Can you compare the calculations without finding the answer?

## Varied Fluency



Use place value counters to complete the number sentences.

352 + 4 tens = \_\_\_\_

Complete:

793 - 60 =	
793 - 70 =	
793 - 80 =	
793 – 90 =	

793 – 60 =
783 - 60 =
773 – 60 =
763 - 60 =
-

733 + 60 =
723 + 60 =
713 + 60 =
703 + 60 =

Complete using <, > or =

$$773 + 1$$
 $\bigcirc$  $773 + 10$  $653 + 10$  $\bigcirc$  $653 - 10$  $647 + 10$  $\bigcirc$  $657 - 10$  $721 + 10$  $\bigcirc$  $653 + 10$ 



## 3-digit & 2-digit Numbers

Spot the Mistake 589 – 70 is equal to 582 Amir What should the answer be?	Amir has subtracted 7 ones instead of 7 tens. The answer should be 519	When I calculated 392 subtract 20 I used my known fact that 9-2=7 Rosie Explain Rosie's method.	Rosie was able to use this fact because 9 tens subtract 2 tens is like doing 9 ones subtract 2 ones. We do not need to subtract any ones or
Write one calculation that could complete all of the statements. 456 - 10 < 466 + 1 > 466 + 0 =	Possible answers include: 496 – 30 406 + 60 416 + 50 (Any calculation with an answer of 466)		hundreds so those columns will stay the same.



# Add 3-digit & 2-digit Numbers

#### Notes and Guidance

Children add multiples of 10, to a 3-digit number with an exchange.

They recognise that when adding tens, it can change the tens and hundreds column. Encourage children to count in tens rather than use column addition.

Draw on knowledge of inverse to work out missing number problems.

### Mathematical Talk

How many tens do we have? How many tens do we need to exchange for 100?

If we know how to count in tens, do we always need to use the column method or other methods?

Would it be easier for us to just count up in our heads?

## Varied Fluency



Miss Wilson has 237 marbles in a box. She adds 8 more bags of 10 marbles. How many marbles does she have now? Write the calculation for this problem.

#### Complete the bar models.





What do you notice?



# Add 3-digit & 2-digit Numbers

Eva and Amir are calculating $783 + 90$ 793, 803, 813, 823, 833, 843, 853, 863, 873 783 + 100 = 883 883 - 10 = 873 Whose method do you prefer? Explain why.	Amir's method is a more efficient method of adding 90. Give children time to discuss each method and try them out with different numbers.	Which is the odd one out? Why? 336 + 80 453 + 60 347 + 70 285 + 80	285 + 80 is the odd one out because in all the others the tens columns add up to 11 tens.
Sort these calculations into two groups. Justify your answer. 257 + 60 70 + 637 40 + 234 20 + 391 Compare your groups with a friend. Are they the same?	Possible ways to sort: Odds and evens Over and under 500 Exchanging and not exchanging		



# Subtract 2-digits from 3-digits

#### Notes and Guidance

Children subtract multiples of 10 from a 3-digit number, with an exchange. The examples show different ways this concept could be taught using number lines and part-whole models.

The column method could be used, however, it is not the most efficient method.

Counting backwards in tens or using 100 to help will support mental strategies.

## Mathematical Talk

How many tens do we exchange one hundred for?

How can we partition 70 to subtract it from 240 more efficiently? Show this on the number line.

Can you model Amir's method using a number line?

## Varied Fluency





## Subtract 2-digits from 3-digits

#### Reasoning and Problem Solving

Complete the missing digits. 13 - 50 = 85 334 - 0 = 294 545 = 65 - 70	135 40 615	How many different methods could you use to solve 837 – 90? Share your methods with a partner.	Possible methods: 837 - 100 = 737 737 + 10 = 747 90 = 37 and 53 (could show in part-whole model) 837 - 37 = 800 800 - 53 = 747
Whitney thinks the rule for the function machine is subtract 60 Is she correct? Explain why.	She is wrong because 567 subtract 60 is 507		837 - 30 = 807 807 - 60 = 747
Input Rule Output 567> 497	The rule is subtract 70		Expanded or formal written methods.

55



## Add & Subtract 100s

#### Notes and Guidance

Children build on their knowledge of adding 100s together e.g. 300 + 500, by adding ones and tens to solve calculations such as 234 + 500

It is important to develop flexibility and ask the children why the column method isn't always the most effective method. Highlight that when adding and subtracting 100s, the ones and tens columns are not affected.

### Mathematical Talk

What do you notice when we add and subtract 100s from a 3-digit number?

Do I need to add or subtract £200 to solve the worded problem? Can you show this on a number line or a bar model?

Is there more than one way to complete the boxes?

# Varied Fluency

Use the place value grid and Base 10 to help you calculate two hundred and thirty-four add three hundred.

Hundreds	Tens	Ones

Eva has saved £675 She saved £200 more than Tommy. How much has Tommy saved?

56

Complete the boxes with a calculation that either adds or subtracts 100s.





## Add & Subtract 100s

306 + 300 = 906 - 300 Alex Is she correct? Explain how you know.	She is correct because both give an answer of 606	Complete the scenarios so they match the bar model. 476 200 676 Ron has altogether. He spends and has £476 pounds	Ron has £676 altogether. He spends £200 and has £476 pounds left. Jack has £476 Eva has £200
Teddy starts with the number 356 He adds a multiple of 100 His new number is greater than 500 but less than 800 Complete the table.	He couldn't have added 100, 500 or 600 but he could have added 200, 300 or 400	left. Jack has Eva has £200 They have altogether.	They have £676 altogether. Amir has £200 more than Rosie. Amir has £676 Rosie has £476 Children will then draw their own bar models to match the numbers they have chosen.
Numbers he couldn't have addedNumbers he could have addedImage: Numbers he could have addedImage: Numbers he could have added		Amir has £200 more than Rosie. Amir has Rosie has Draw your own bar model where one of the parts is a multiple of 100 Write scenarios to match the bar model.	



## **Pattern Spotting**

#### Notes and Guidance

Children consolidate adding ones, tens and hundreds to 3digit numbers.

Drawing the previous steps together, children look for patterns between calculations to enable them to predict answers and to develop their number sense.

Ensure children reflect on the similarities and differences between calculations to highlight the patterns.

#### Mathematical Talk

What do you notice? Which strategy can we use to add these numbers?

Do we need to write a zero in the hundreds column when there are no hundreds left?

If I know 7 + 8 = 15, what else do I know?

### Varied Fluency

What has happened to each starting number? How do you know?

Be	fore	After	
Three	nundred	Three hundred	4
and	J forty	and seventy	
Calculate:			
253 + 2	253 + 20	25	3 + 200
253 — 2	253 — 20	25	3 – 200

What is the same and what is different about each calculation?

If we know 250 + 40 = 290, what else do we know? Show your findings in part-whole models or bar models and write number sentences to match.



# **Pattern Spotting**

## **Reasoning and Problem Solving**

Dora uses column addition to solve 251 + 4



Is this the most efficient method?

Explain what Dora could have done.

Tell Dora how she can use your strategy to solve 241 + 40 and 241 + 400

The best strategy is to complete 1 + 4, which is 5 and the 2 hundreds and 5 tens stay the same.

When adding 40 it is the tens column which Dora needs to look at because 40 is 4 tens.

When adding 400, she needs to look at the hundreds column because 400 is 4 hundreds.

#### Investigate

Does adding and subtracting ones to a 3-digit number only affect the ones column?

Does adding and subtracting tens to a 3-digit number only affect the tens column?

No, the ones can change the ones column and any column to the left e.g. 123 + 9 and 402 - 4The tens column can change itself and the hundreds. column e.g. 456 + 50 and 456 - 60When adding and subtracting from any column, it can only affect its own column and columns to the left.



C A

## Add 2-digit Numbers (2)

### Notes and Guidance

Children use Base 10 and partitioning to add together 2digit numbers including an exchange. They could be encouraged to draw the Base 10 alongside recording any formal column method.

They have already seen what happens when there are more than 10 ones and should be confident in exchanging 10 ones for one 10.

### Mathematical Talk

Can you represent the ones and tens using Base 10? What is the value of the digits? How many ones do we have altogether? How many tens do we have altogether? Can we exchange ten ones for one ten? What is the sum of the numbers? What is the total? How many have we got altogether?

#### Varied Fluency

	64	+	17	=	
--	----	---	----	---	--

	04
4 ones + 7 ones =	<u>+ 1 7</u>
	11
6  tens + 1  ten =	<u>+ 70</u>
	81

\_\_\_\_\_ tens + \_\_\_\_\_ ones = \_\_\_\_

#### Find the sum of 35 and 26

- ||| ÷
- Partition both the numbers.
- Add together the ones. Have we got 10 ones?
- Exchange 10 ones for 1 ten.
- How many ones do we have?
- Add together the tens. How many dowe have altogether?

Class 3 has 37 pencils. Class 4 has 43 pencils.

How many pencils do they have altogether?



13 + 29

19 + 23

14 + 28

18 + 24

15 + 27

17 + 25

16 + 26

All the pairs of

ones add up to 12

# Add 2-digit Numbers (2)

#### Reasoning and Problem Solving

Can you create a calculation where there will be an exchange in the ones and your answer will have two ones and be less than 100?	There are lots of possible solutions. E.g. 33 + 29 = 62	Find all the possible pairs of numbers that can complete the addition.
How many different ways can you solve 19 + 11? Explain your method to a partner.	Children might add the ones and then the tens. Children should	$\frac{+2}{42}$
Use concrete or pictorial resources to help explain your method.	notice that 1 and 9 are a number bond to 10 which makes the calculation easier to complete mentally.	How do you know you have found all the pairs? What is the same about all the pairs of numbers?

61



# Subtract with 2-digits (2)

#### **Notes and Guidance**

Children use their knowledge that one ten is the same as ten ones to exchange when crossing a ten in subtraction.

Continue to use concrete manipulatives (such as Base 10) and pictorial representations (such as number lines and partwhole models) to develop the children's understanding.

The skill of flexible partitioning is useful here when the children are calculating with exchanges.

## Mathematical Talk

Have we got enough ones to take away?

- Can we exchange one ten for ten ones?
- How many have we got left?
- What is the difference between the numbers?
- Do we always need to subtract the ones first? Why do we always subtract the ones first?
- Which method is the most efficient to find the difference, subtraction or counting on?

## Varied Fluency

Use the number line to subtract 12 from 51

51

Can you subtract the ones first and then the tens? Can you partition the ones to count back to the next ten and then subtract the tens?

42

30

20

42 - 15 =

-10

42 40

- partition
- We can't subtract the ones. Can we differently?

Now we can subtract the ones and then subtract the tens. 42 - 15 = 277









# Subtract with 2-digits (2)





2-digit & 3-digit Numbers				
Notes and Guidance	Varied Flu	ency		
Children focus on the position of numbers and place value to add and subtract 2-digit and 3-digit numbers.	Aatch the calculation	n to the correct repres	entation and	solve.
They represent numbers using Base 10 and line up the place	26 + 461			
value columns.			_	
in this step, children add hombers without an exchange.	553 — 32	H		
Mathematical Talk				•
Where would these digits go on the place value chart? Why?	544 + 22	H	T	O ■∮∮
When we subtract, why do we not make both numbers? Why do we make both numbers when weadd?	Represent the calcula 388 – 44	ations using Base 10 a 167 + 32	and solve the 265	m. 5 — 43
What is the same about the additions and subtractions? What changes?	Calculate: 365 <u>+ 23</u> <u>–</u>	365 365 23 <u>+ 32</u>	3	65 <u>32</u>

64



## 2-digit & 3-digit Numbers





# Add 2-digit & 3-digit Numbers

#### Notes and Guidance

Children deepen their understanding of adding 2-digit and 3digit numbers in this step. They start adding numbers where there is an exchange from ones to tens, they then move on to exchanging tens to hundreds before adding numbers where there are exchanges in both columns.

Highlight the links between the concrete representations and the column method to support children in understanding how the column method works.

#### Mathematical Talk

What happens when we have 10 ones in a column? How many tens do we exchange 10 ones for? How do we show the exchange in the column method?

What happens when we have 10 tens in a column? How many hundreds do we exchange 10 tens for? How do we show the exchange in the column method?

What do you notice about the additions in the models? How many exchanges do we need to make?

# Varied Fluency





# Add 2-digit & 3-digit Numbers

# Reasoning and Problem Solving



Here is her working out:

	2	6	5	
+		2	7	
	2	8	2	

Is she correct? Explain why.

Eva is incorrect because she has not exchanged ten ones for one ten or shown this in the column method.

She should have added an extra ten to the tens column. The correct answer is 292

Sort the addit	No exchange:		
No exchange	Exchange 10 ones	Exchange 10 tens	342 + 35
		Exchange 10 ones 375 + 18 456 + 27	
375 + 18 910 + 79	456 + 72 456 + 27	912 + 79 Exchange 10 tens	
Can you write column?	456 + 72		
Choose one 2 number. Write addition the ones and t	23 + 487 35 + 467 56 + 756 619 + 81		
23 35 81 5			



# Subtract 2-digits from 3-digits

#### Notes and Guidance

Children focus on the position of numbers and place value to subtract 2-digits from 3-digits using the column method. Children start by exchanging one ten for ten ones. Next they exchange one hundred for ten tens before subtracting numbers where there are exchanges in both columns. Encourage children to use Base 10 and place value counters so they can physically exchange and see the link between the concrete and the written column method.

### Mathematical Talk

How does the concrete representation match the written column method?

How do you know that you need to exchange?

What do you notice about the subtractions to find the missing numbers? How many exchanges are there?

## Varied Fluency



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## Subtract 2-digits from 3-digits

Rosie th	inks 3	352 -	- 89 :	= 33	7	Rosie is incorrect because she has	Alex, Teddy and Dora are trying to work out 300 – 57	Accept different answers as long as
		н	Т	0		subtracted the	Who has the most efficient way of	they are justified.
		3	5	2		order instead of	working it out?	even suggest
	-		8	9		exchanging.		subtracting 60
		3	3	7		The answer should	means difference, so I	and then adding 3
ls she co Explain	orrect why.	?				00 200	Alex 56 and get the right answer.	
Use <, > correct.	> or = 234 - 472 - 406 -	= to r - 47 - 84 - 89	nake	the s 234 473 416	tatements 57 84 99	> < =	I can count on from 57 to 100, and then count on to 300 I can use the column method to work it out and exchange when I need to.	



# Add Two 3-digit Numbers (1)

#### Notes and Guidance

Children add two 3-digit numbers with no exchange. They should focus on the lining up of the digits and setting the additions clearly out in columns.

Having exchanged between columns in recent steps, look out for children who exchange ones and tens when they don't need to.

Reinforce that we only exchange when there are 10 or more in a column.

### Mathematical Talk

Where would these digits go on the place value chart? Why?

Why do we make both numbers when we add?

- Can you represent \_\_\_\_\_using the equipment?
- Can you draw a picture to represent this?

Why is it important to put the digits in the correct column?

# Varied Fluency

Complete the calculations.



Н	Т	0	
100 000	10 10 10 10 10		+=
000 000 000 100 000	10 10 10 10		

- Use the column method to calculate:
  - Three hundred and forty-five add two hundred and thirty-six.
  - Five hundred and sixteen plus three hundred and sixty-two.
  - The total of two hundred and forty-seven and four hundred and two.



## Add Two 3-digit Numbers (1)

### **Reasoning and Problem Solving**

#### Jack is calculating 506 + 243

Here is his working out.

		5	6
+	2	4	3
	2	9	9

Can you spot Jack's mistake? Work out the correct answer. Jack hasn't used zero as a place holder in the tens column. The correct answer should be 749

Here are three digit cards.



Alex and Teddy are making 3-digit numbers using each card once.



Work out the total of their two numbers.

Alex's number is 432 Teddy's number is 234

The total is 666



# Add Two 3-digit Numbers (2)

### Notes and Guidance

Children add two 3-digit numbers with an exchange. They start by adding numbers where there is one exchange required before looking at questions where they need to exchange in two different columns. Children may use Base 10 or place value counters to model their understanding. Ensure that children continue to show the written method alongside the concrete so they understand when and why an exchange takes place.

## Mathematical Talk

How many ones do we need to exchange for one ten?

How many tens do we need to exchange for one hundred?

Can you work out how many points Eva and Ron scored each over the two games?

Why is it so important to show the exchanged digit on the column method?

# Varied Fluency

Use place value counters to calculate 455 + 436

Н	Т	0		4	<b>_</b>	_
100 100 100 100	10 10 10			4	5	5
	10 10		+	4	3	6
100 100 100 100	10 10 10					

Eva and Ron are playing a game. Eva scores 351 points and Ron scores 478 points. How many points do they score altogether? How many more points does Ron score than Eva?

Eva and Ron play the game again. Eva scores 281 points, Ron scores 60 less than Eva. How many points do they score altogether?


=



# Add Two 3-digit Numbers (2)

# Reasoning and Problem Solving

#### Roll a 1 to 6 die. Fill in a box each time you roll.

+

#### Can you make the total:

- An odd number
- An even number
- A multiple of 5
- The greatest possible number
- The smallest possible number

Discuss the rules with the children and what they would need to roll to get them e.g. to get an odd number only one of the ones should be odd because if both ones have an odd number, their total will be even.





Explain why you do not have to work out the answers to compare them.

< = 590

In the first one we start with the same number, so the one we add more to will be greater. In the second 325 is one less than 326 and 259 is one more than 258, so the total will be the same. In the last one 401 is 10 more than 391, so we need to add 10 less than 600.



### Subtract 3-digits from 3-digits (1)

#### Notes and Guidance

It is important for the children to understand that there are different methods of subtraction. They need to explore efficient strategies for subtraction, including:

- counting on (number lines)
- near subtraction
- number bonds

They then move on to setting out formal column subtraction supported by practical equipment.

#### Mathematical Talk

Which strategy would you use and why?

How could you check your answer is correct?

Does it matter which number is at the top of the subtraction?

# Varied Fluency

We can count on using a number line to find the missing value on the bar model. E.g.





Use this method to find the missing values.

390	
273	?

294	
?	134

- There are 146 girls and boys in a swimming club. 115 of them are girls. How many are boys?
- Mo uses Base 10 to subtract 142 from 373



	3	7	3
-	1	4	2

Use Mo's method to calculate:

565 - 154 565 - 145 565 - 165



# Subtract 3-digits from 3-digits (1)

## **Reasoning and Problem Solving**

Start with the number 888 Roll a 1-6 die three times, to make a 3digit number. Subtract the number from 888 What number have you got now?

What's the smallest possible difference?

What's the largest possible difference?

What if all the digits have to be different?

Will you ever find a difference that is a multiple of 10? Why?

Do you have more odd or even differences?

The smallest difference is 222 from rolling 111

The largest difference is 777 from rolling 666

Children will never have a multiple of 10 because you can't roll an 8 to subtract 8 ones.

Children may investigate what is subtracted in the ones column to make odd and even numbers. Use the digit cards to complete the calculation.





Possible answers include:

987 - 647 = 340

The digits in the shaded boxes are odd.

Is there more than one answer?



#### Subtract 3-digits from 3-digits (2)

#### Notes and Guidance

Children explore column subtraction using concrete manipulatives. It is important to show the column method alongside so that children make the connection to the abstract method and so understand what is happening. Children progress from an exchange in one column, to an exchange in two columns. Reinforce the importance of recording any exchanges clearly in the written method.

## Mathematical Talk

Which method would you use for this calculation and why?

What happens when you can't subtract 9 ones from 7 ones? What do we need to do?

How would you teach somebody else to use column subtraction with exchange?

#### Why do we exchange? When do we exchange?

# Varied Fluency

Complete the calculations using place value counters.

372 - 145



629 – 483



0

4

5



Complete the column subtractions showing any exchanges.

Η

2

1

Т

3

9

	Н	Т	0	
	6	8	3	
_	2	3	4	

	Н	Т	0
	5	0	7
_	4	5	1



# Subtract 3-digits from 3-digits (2)

#### Reasoning and Problem Solving

Work out the missing digits.	533 - 218 = 315	Eva is working out 406 — 289	Eva has exchanged from
H T O	504 - 258 = 246	Here is her working out:	the hundred
5 ? 3		Step 1 Step 2	column to the ones so there are
- 2 1 8		$\frac{3}{4}0^{1}6$ $\frac{2}{4}0^{1}6$	106 ones in the
3 1 5		-289 -289	ones column. She should have
		7 027	exchanged 1
H T O			hundred for 10 tens and then 1 ten
? 0 ?		Explain her mistake.	for 10 ones.
- 2 ? 8			406 - 289 = 117
2 4 6		What should the answer be?	

77



#### **Estimate Answers**

#### Notes and Guidance

Children check how reasonable their answers are. While rounding is not formally introduced until Year 4, it is helpful that children can refer to 'near numbers' to see whether an estimate is sensible.

Discuss why estimations are important. Consider real life situations where children or adults need to estimate. Encourage children to estimate calculations before working out precisely to help to check working.

## Mathematical Talk

What would you estimate this to be?

Why did you choose this number?

Why is/isn't this a sensible estimation to an answer?

How does estimating answers help us in real life?

# Varied Fluency

Estimate the position of arrows A and B on the number line. Use your estimations to estimate the difference between A and B.







Use the near numbers to estimate the answers to the calculations:

497 + 304	304 — 27	27 + 52 + 304
27 + 304	497 — 52	304 — 52 — 27
52 + 497	497 — 304	304 + 52 - 27



#### **Estimate Answers**

## Reasoning and Problem Solving



I estimate 143 — 95 will be 50 because I will subtract 100 from 150

Is this a good estimate? Why?

Are there any other ways he could have estimated?

Yes, because he found two numbers close to the original numbers.

He could have rounded to the nearest 10 and calculated.

140 - 100 (= 40)

Use the number cards to make different Possible answers: calculations with an estimated answer of 70 121 - 48(120 - 50)33 48 41 121 41 + 33(40 + 30)328 255 398 398 - 328 (400 - 330)



#### **Check Answers**

## Notes and Guidance

Children explore ways of checking to see if an answer is reasonable.

Checking using inverse is to be encouraged so that children are using a different method and not just potentially repeating an error, for example, if they add in a different order.

# Mathematical Talk

How can you tell if your answer is sensible?

Does knowing if a number is close to a multiple of 100 help when adding and subtracting 3-digit numbers? How does it help?

Does it help to check your answer if you spot which numbers are near to multiples of 10?

# Varied Fluency

Use a subtraction to check the answer to the addition.

Alex has baked 145 cakes for a bun sale. She sells 78 cakes. How many does she have left?

Show your answer using a bar model and check your answer using an addition.



660 120 540 + - =

How does counting in 10s, 50s and 100s help?



#### **Check Answers**

#### Reasoning and Problem Solving

Mo

(0)0

If I add two numbers together, I can check my answer by using a subtraction of the same numbers after e.g. to check 23 + 14, I can do 14 - 23

Do you agree? Explain why.

No, because you cannot have "part subtract part".

You need to find the whole and this needs to be at the start of the subtraction then you subtract a part to check the remaining part. I completed an addition and then used the inverse to check my calculation. When I checked my calculation, the answer was 250. One of the other numbers was 355. What could the calculation be?



Possible answers: 355 - 105 = 250 605 - 355 = 250So the calculation could have been: 250 + 105 = 355 250 + 355 =605



#### Year 3 | Autumn Term | Week 9 to 12 – Number: Multiplication & Division

# Overview

Small Steps

Multiplication – equal groups	
Multiplication using the symbol	R
Using arrays	R
2 times-table	R
5 times-table	R
Make equal groups - sharing	R
Make equal groups - grouping	R
Divide by 2	R
Divide by 5	R
Divide by 10	R
Multiply by 3	
Divide by 3	
The 3 times table	

# Notes for 2020/21

Children should have met the 2, 5 and 10 times table including being able to divide by 2, 5 and 10. However it may not be fully embedded.

These recap steps could be filtered in during starters or morning work to aim for fluency.



#### Year 3 | Autumn Term | Week 9 to 12 – Number: Multiplication & Division

# Overview

Small Steps

# Notes for 2020/21









## Multiplication – Equal Groups

#### Notes and Guidance

Children recap their understanding of recognising, making and adding equal groups. This will allow them to build on prior learning and prepare them for the next small steps.

## Mathematical Talk

What is the same and what is different between each of the groups?

What does the 3 represent?

What does the 8 represent?

How can we represent the groups?

# Varied Fluency



How many different ways can you represent: Six equal groups with 4 in each group? Six 4s?

#### Complete:

	Add It
Say it	Multiply it
There are equal groups with in each group. There are altogether.	



## Multiplication – Equal Groups

## **Reasoning and Problem Solving**



#### Year 2 | Autumn Term | Week 11 to 12 - Number: Multiplication & Division

# The Multiplication Symbol

# Notes and Guidance

Children are introduced to the multiplication symbol for the first time. They should link repeated addition and

multiplication together, using stem sentences to support their understanding.

They should also be able to interpret mathematical stories and create their own involving multiplication.

The use of concrete resources and pictorial representations is still vital for understanding.

# Mathematical Talk

What does the 3 represent? What does the 6 represent?

What does 'lots of' mean?

Does  $18 = 3 \times 6$  mean the same?

How is 6 + 6 + 6 the same as  $3 \times 6$ ? How is it different?

#### Varied Fluency

Complete the sentences to describe the equal groups.



Complete:

Three 2s	Draw It	Addition	Multiplication
There are 3 equal groups with 2 in each group.			

#### Complete:

Addition	Multiplication	Story
10 + 10 + 10		
	6×5	



= 18

× = 18



R

## The Multiplication Symbol

#### **Reasoning and Problem Solving**

3 + 3 + 3 = 3 × 3	He is correct because 3 + 3 + 3 = 9 and $3 \times 3 = 9$	Think of a multiplication to complete: $6 + 6 + 6 > \ × \$	Any two numbers which multiply together to give an answer of less than 18
Is Mo correct? Explain why. Draw an image to help you.		The total is 12, what could the addition and multiplication be?	$6 + 6 = 2 \times 6$ 2 + 2 + 2 + 2 + 2 + 2
Use <, > or = to make the statements correct.	3 × 5 < 5 + 5 + 5 + 5		$= 0 \times 2$ 3 + 3 + 3 + 3 = 4 × 3 4 + 4 + 4 = 3 × 4
$3 \times 5$ $5 + 5 + 5 + 5$ $2 \times 2$ $2 + 2$ $10 \times 2$ $5 + 5 + 5$	$2 \times 2 = 2 + 2$ 10 × 2 > 5 + 5 + 5		$12 = 1 \times 12$ 1+1+1+1+1+1+ 1+1+1+1+1=12 ×1



## **Use Arrays**

#### Notes and Guidance

Children explore arrays to see the commutativity of multiplication facts e.g.  $5 \times 2 = 2 \times 5$ 

The use of the array could be used to help children calculate multiplication statements.

The multiplication symbol and language of 'lots of' should be used interchangeably.

## Mathematical Talk

Where are the 2 lots of 3?

Where are the 3 lots of 2?

What do you notice?

What can we use to represent the eggs?

Can you draw an image?

#### Varied Fluency

The image, find 2 imes 5 and 5 imes 2



Can you represent this array using another object?

Complete the number sentences to describe the arrays.





\_ × \_\_\_\_ and \_\_\_\_ × \_\_\_\_



 $4 \times 5 = 5 \times 4$ 

3 lots of 10 = 10 lots of 3

#### Year 2 | Autumn Term | Week 11 to 12 – Number: Multiplication & Division

#### **Use Arrays**

#### **Reasoning and Problem Solving**

With 12 cubes, how many different arrays can you create?		Find different ways to solve six lots of three.	Count in 3s 3 lots of 3 add 3 lots of 3 5 × 3 add 1 × 3
Once you have created your array complete:	$1 \times 12 = 12 \times 1$ $2 \times 6 = 6 \times 2$ $3 \times 4 = 4 \times 3$		etc.
	0 / 1 - 1 / 0	Part of this array is hidden.	
			4 × 2 5 × 2 6 × 2 7 × 2
		The total is less than 16	
		What could the array be?	

90



R



## The 2 Times-Table

#### Notes and Guidance

Children should be comfortable with the concept of multiplication so they can apply this to multiplication tables.

Images, as well as number tracks, should be used to encourage children to count in twos.

Resources such as cubes and number pieces are important for children to explore equal groups within the 2 times-table.

#### Mathematical Talk

If 16 p is made using 2 p coins, how many coins would there be?

How many 2s go into 16?

How can the images of the 5 bicycles help you to solve the problems?

## Varied Fluency







Complete the number track.







If there are 14 wheels, how many bicycles are there?



R

#### The 2 Times-Table

#### Reasoning and Problem Solving

Fill in the blanks. $3 \times \underline{} = 6$ $\underline{} \times 2 = 20$ $\underline{} = 8 \times 2$	2 10 16	Eva says, Every number in the 2 times-table is even.	Yes, because 2 is even, and the 2 times-table is going up in 2s. When you add two even numbers the answer is always
Tommy says that 10 × 2 = 22 Is he correct? Explain how you know.	No Tommy is wrong because 10 $\times 2 = 20$ Children could draw an array or a picture to explain their answer.	Is she correct? Explain your answer.	even.

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#### Year 2 | Autumn Term | Week 11 to 12 – Number: Multiplication & Division

# The 5 Times-Table

#### Notes and Guidance

Children can already count in 5s from any given number. They will also have developed understanding of the 2 timestable.

This small step is focused on the 5 times table and it is important to include the use of zero. Children should see the = sign at both ends of the calculation to understand that it means 'equals to'.

## Mathematical Talk

If there are 30 petals, how many flowers? Can you count in 5s to 30? How many 5s go into 30?

How many 5s go into 35?

What does each symbol mean?

#### Varied Fluency

- How many petals altogether?
  - Write the calculation.
- There are 35 fingers. How many hands?













### The 5 Times-Table

## Reasoning and Problem Solving

Is Mo correct? Every number in the 5 times table is odd.	Mo is incorrect because some of the multiples of the five times- table are even, e.g. 10, 20, 30	Tommy and Rosie have both drawn bar models to show 7 $\times$ 5 5555555	The total shown is the same. Tommy's bar shows seven lots of 5 whereas Rosie's bar show
Explain your answer.		35           7         7         7         7	five lots of 7 Children can
Tubes of tennis balls come in packs of 2 and 5	Whitney could have:	What's the same and what is different about their bar models?	choose either way to represent $4 \times 5$
Whitney has 22 tubes of balls.	4 packs of 5 and 1 pack of 2,	Draw your own bar model to represent $4 \times 5$	
How many of each pack could she have?	11 packs of 2 and O packs of 5, 2 packs of 5 and 6		
How many ways can you do it?	packs of 2		



# Make Equal Groups - Sharing

#### Notes and Guidance

Children divide by sharing objects into equal groups using one-to-one correspondence. They need to do this using concrete manipulatives in different contexts, then move on to pictorial representations.

Children will be introduced to the ' $\div$ ' symbol. They will begin to see the link between division and multiplication.

# Mathematical Talk

How many do you have to begin with? How many equal groups are you sharing between? How many are in each group? How do you know that you have shared the objects equally?

\_\_\_\_ has been shared equally into \_\_\_\_ equal groups.
I have \_\_\_\_ in each group.
\_\_\_\_ groups of \_\_\_\_ make \_\_\_\_

## Varied Fluency



There are	_ cubes altogether.
There are	_boxes.
There are	_ cubes in each box.



Can you share the 12 cubes equally into 3 boxes?

24 children are put into 4 equal teams. How many children are in each team?

Can you use manipulatives to represent the children to show how you found your answer?

Ron draws this bar model to divide 20 into 4 equal groups.
How does his model represent this?
He writes 20 ÷ 4 = 5



What other number sentences could Ron create using his model?



# Make Equal Groups - Sharing

# **Reasoning and Problem Solving**

#### Jack says,



I can work out 40 ÷ 2 easily because I know that 40 is the same as 4

tens.

This is what he does:



Is it possible to work out 60 ÷ 3 in the same way? Prove it.

Is it possible to work out  $60 \div 4$ ? What is different about this calculation?

#### Possible answer :



For  $60 \div 4$  the children will need to exchange 2 tens for 20 ones so they can put one 10 and 5 ones into each group.

 Alex has 20 sweets and shares them between 5 friends.

Tommy has 20 sweets and shares them between 10 friends.

Whose friends will receive the most sweets?

How do you know?

Alex's friends get more because Tommy is sharing with more people so they will get fewer sweets each. Alex's friends will get 4 sweets each whereas Tommy's friends will only get 2 sweets each.



## Make Equal Groups - Grouping

#### Notes and Guidance

Children divide by making equal groups. They then count on to find the total number of groups.

They need to do this using concrete manipulatives and pictorially in a variety of contexts.

They need to recognise the link between division, multiplication and repeated addition.

## Mathematical Talk

How many do you have to begin with? How many are in each group? How many groups can you make?

How long should your number line be? What will you count up in?

\_\_\_\_ groups of \_\_\_\_\_ make \_\_\_\_

#### Varied Fluency

Pencils come in packs of 20We need to put 5 in each pot.How many pots will we need?

There are \_\_\_\_ pencils altogether. There are \_\_\_\_ pencils in each pot. There are \_\_\_\_ pots.

<sup>7</sup> Mrs Green has 18 sweets. She puts 3 sweets in each bag. How many bags can she fill?

18



 $\boxed{18} \div \boxed{3} = \boxed{}$ 





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Mo uses a number line to work out how many equal groups of 2 he can make from 12



Use a number line to work out how many equal groups of 5 you can make from 30



# Make Equal Groups - Grouping

# **Reasoning and Problem Solving**

You have 30 counters.



How many different ways can you put them into equal groups?

Write down all the possible ways.

10 groups of 3 3 groups of 10 6 groups of 5 5 groups of 6 2 groups of 15 15 groups of 2 1 group of 30 30 groups of 1 Amir has some counters. He makes 5 equal groups.

The amount he started with is greater than 10 but less than 35

How many counters could he have started with?

How many will be in each group?



He could have 30 counters in 5 groups of 6

25 counters in 5 groups of 5

20 counters in 5 groups of 4

15 counters in 5 groups of 3



#### Notes and Guidance

Children should be secure with grouping and sharing. They will use this knowledge to help them divide by 2

They will be secure with representing division as an abstract number sentence using the division and equals symbol.

Children should be able to count in 2s and know their 2 times table.

# Mathematical Talk

What do you notice when you group these objects into twos?

Is there a link between dividing by 2 and halving?

What is different about sharing into two groups and grouping in twos?

Can we write a multiplication sentence as well as a division sentence? What do you notice?

#### Varied Fluency

Complete the stem sentences.

I have \_\_\_\_ cubes altogether. There are \_\_\_\_ in each group. There are \_\_\_\_ groups.

Group the socks into pairs.

Complete the number sentences.

,,,,,,,,,,,

- Mo and Tommy have 12 sweets between them. They share them equally. How many sweets does each child get?
  - There are \_\_\_\_ sweets altogether. There are \_\_\_\_ groups. There are \_\_\_\_ in each group.



Complete the bar model and write a calculation to match.



## **Reasoning and Problem Solving**

I have 24p. I divide it equally between 2 friends. How much will they get each?

I have 24p in 2p coins. How many 2p coins do I have?

Consider the two questions above. What is the same and what is different?

Tommy and Annie have some counters.

Tommy shares his counters into 2 equal groups. He has 15 in each group.

Annie groups her counters in twos. She has 19 groups.

Who has more counters and by how many? How did you work it out?

The calculation is the same in both In the first question we are sharing, whereas in the second question we are grouping. Tommy has 30 counters. Annie has 38 counters. Annie has 8 more. Children could have compared 15 and 19 and realised they could have done  $2 \times 4$ 

Ron has shared some grapes equally between two friends.



Ron's friends

Each friend receives fewer than 50 grapes.

Complete the sentences to describe the number of grapes Ron started with.

He must have started with...

He could have started with...

He can't have started with...

Possible answer:

He must have started with an even number of grapes.

He could have started with 40 grapes.

He can't have started with 100 grapes.



## Notes and Guidance

During this step, children focus on efficient strategies and whether they should use grouping or sharing depending on the context of the question.

They use their knowledge of the five times table to help them divide by  $\mathbf{5}$ 

They will continue to see the = sign both before and after the calculation.

# Mathematical Talk

How can we represent the problem using objects/images?

How does knowing your 5 times table help when dividing by 5?

Circle all the multiples of 5 on a 100 square. What do you notice about the numbers? Can you explain the pattern? How does this help you to divide these numbers?

When would we count in 5s?

#### Varied Fluency

🕇 Take 30 cubes.

How many towers of 5 can you make? You can make \_\_\_\_ towers of 5 \_\_\_\_ towers of 5 is the same as 30 30 is the same as \_\_\_ towers of 5



🔰 40 pencils are shared between 5 children.



÷ = =

How many pencils does each child get?

- Group the 1p coins into 5s.
   How many 5p coins do we
   need to make the same amount of money?
   Draw coins and complete the missing information.
  - lots of 5p = 20 one pence coins
  - \_\_\_\_ lots of 5p = 20p
  - 20p = \_\_\_ × 5p
  - 20p ÷ 5 = \_\_\_

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#### Reasoning and Problem Solving

A party bag contains 5 sweets. A jar contains 5 party bags.



Ron has 75 sweets.

How many party bags will he need?

How many jars will he need?

15 party bags. 3 jars.

multiplication and division sentences. How many can you make?

Use the number cards to make

20 5 10 4  $4 \times 5 = 20$   $5 \times 4 = 20$   $20 \div 4 = 5$   $20 \div 5 = 4$   $5 \times 2 = 10$   $2 \times 5 = 10$   $10 \div 2 = 5$   $10 \div 5 = 2$   $20 \div 2 = 10$   $20 \div 10 = 2$   $2 \times 10 = 20$  $10 \times 2 = 20$ 





## **Notes and Guidance**

Children should already be able to multiply by 10 and recognise multiples of 10. They will need to use both grouping and sharing to divide by 10 depending on the context of the problem.

Children start to see that grouping and counting in 10s is more efficient than sharing into 10 equal groups.

## Mathematical Talk

What can we use to represent the problem?

How does knowing your 10 times table help you to divide by 10?

Circle all the multiples of 10 on a hundred square. What do you notice? Can you explain the pattern?

How many groups of 10 are there in \_\_\_\_?

#### Varied Fluency

Apples can be sold in packs of 10 How many packs can be made below?



When 30 apples are sold in packs of 10, \_\_\_\_ packs of apples can be made.

Can you show this in a bar model?

Label and explain what each part represents.

- I have 70p in my pocket made up of 10p coins. How many coins do I have? Draw a picture to prove your answer.
- Fill in the missing numbers.
  - 70 ÷ 10 = \_\_\_
  - 6 tens  $\div$  1 ten =
  - $5 = \div 10$
  - There are <u>tens</u> in 40



#### **Reasoning and Problem Solving**

Mrs Owen has some sweets.

She shares them equally between 10 tables.

How many sweets could each table have?

Find as many ways as you can.

What do you notice about your answers?

#### True or false?

Dividing by 10 is the same as dividing by 5 then dividing by 2

They could have:  $10 \div 10 = 1$   $20 \div 10 = 2$   $30 \div 10 = 3$   $40 \div 10 = 4$   $50 \div 10 = 5$ etc

The tens digit is the same as the answer.

True

Cakes are sold in boxes of 10 Jack and Alex are trying to pack these cakes into boxes.



Who is correct? Explain how you know.

Alex is correct because there are 60 cakes and 60 divided by 10 is 6

Jack has incorrectly grouped the cakes, he might have counted the rows wrong. He hasn't put them in 10s. He incorrectly assumed there were 10 in each row.



# Multiply by 3

## Notes and Guidance

Children draw on their knowledge of counting in threes in order to start to multiply by 3

They use their knowledge of equal groups to use concrete and pictorial methods to solve questions and problems involving multiplying by 3

Mathematical Talk

How many equal groups do we have?

How many are in each group?

How many do we have altogether?

- Can you write a number sentence to show this?
- Can you represent the problem in a picture?
- Can you use concrete apparatus to solve the problem?
- How many lots of 3 do we have?
- How many groups of 3 do we have?

# Varied Fluency

There are five towers with 3 cubes in each tower. How many cubes are there altogether?

# \_\_\_\_×\_\_\_\_=\_\_\_

There are 7 tricycles in a playground. How many wheels are there altogether? Complete the bar model to find the answer.





\_\_\_\_ lots of \_\_\_\_ = \_\_\_\_

X



# Multiply by 3

### Reasoning and Problem Solving

There are 8 children. Each child has 3 sweets. How many sweets altogether?

Use concrete or pictorial representations to show this problem.

Write another repeated addition and multiplication problem and ask a friend to represent it. There are 24 sweets altogether. Children may use items such as counters or cubes. They could draw a bar model for a pictorial representation.

 $5 \times 3 + 3$ If  $5 \times 3 = 15$ , which number sentences would find the answer to  $6 \times 3$ ? because one more lot of 3 will find  $5 \times 3 + 6$ • the answer.  $5 \times 3 + 3$ • 15 + 3 because 15 + 3• adding one more 15 + 6٠ lot of 3 to the answer to 5 lots  $3 \times 6$ • will give me 6 lots. Explain how you know. 3 x 6 because 3  $\times 6 = 6 \times 3$ (because multiplication is commutative).



## Notes and Guidance

Children explore dividing by 3 through sharing into three equal groups and grouping in threes.

They use concrete and pictorial representations and use their knowledge of the inverse to check their answers.

## Mathematical Talk

Can you put the counters into groups of three?

- Can you share the number into three groups?
- What is the difference between sharing and grouping?

# Varied Fluency

Circle the counters in groups of 3 and complete the division.



Circle the counters in 3 equal groups and complete the division.

÷3= 000000

What's different about the ways you have circled the counters?

There are 12 pieces of fruit. They are shared equally between 3 bowls. How many pieces of fruit are in each bowl? Use cubes/counters to represent fruit and share between 3 circles.





Bobbles come in packs of 3 If there are 21 bobbles altogether, how many packs are there?



### Reasoning and Problem Solving

Share 33 cubes between 3 groups.

#### Complete:

There are 3 groups with \_\_\_\_\_ cubes in each group.  $33 \div 3 = \____$ 

Put 33 cubes into groups of 3

#### Complete:

There are \_\_\_\_\_ groups with 3 cubes in each group.  $33 \div 3 =$ \_\_\_\_

What is the same about these two divisions? What is different?

The number sentences are both the same. The numbers in each number sentence mean different things. In the first question, the '3' means the number of groups the cubes are shared into because the cubes are being shared. In the second question, the '3' means the size of each group.

#### Jack has 18 seeds.

He plants 3 seeds in each pot.

Which bar model matches the problem?





Explain your choice.

#### Bar model B

matches the problem because Jack plants 3 seeds in each pot, therefore he will have 6 groups (pots), each with 3 seeds.


#### The 3 Times Table

#### **Notes and Guidance**

Children draw together their knowledge of multiplying and dividing by three in order to become more fluent in the three times table.

Children apply their knowledge to different contexts.

#### Mathematical Talk

Can you use concrete or pictorial representations to help you?

What other facts can you link to this one?

What other times table will help us with this question?

# Varied Fluency

Complete the number sentences.

1 triangle has 3 sides. 3 triangles have \_\_\_\_\_ sides in total. \_\_\_\_\_ triangles have 6 sides in total. 5 triangles have \_\_\_\_\_ sides in total.



Tick the number sentences that the image shows.

	$12 \div 3 = 4$ $12 = 4 \times 3$ $3 \div 4 = 12$	$3 = 12 \div 4$ $3 \times 12 = 4$ $3 \times 4 = 12$



1 × 3 =	× 3 = 30
2 × = 6	8 × = 24
$_{=} 3 \times 3$	6 × 3 =
9 × 3 =	$21 = \times 3$



# The 3 Times Table

# Reasoning and Problem Solving

Sort the cards below so they follow round Order: in a loop.

Start at 18 – 3

Calculate the answer to this calculation. The next card needs to be begin with this answer.



18 — 3
15 ÷ 3
5 × 2
10 × 2
20 + 1
21 ÷ 3
7 × 2
14 — 2
12 ÷ 3
4 × 2
8 — 5
3×6

Start this rhythm:	Clicks are
Clap, clap, click, clap, clap, click.	multiples of three. On the 15th beat, I
Carry on the rhythm, what will you do on the 15th beat?	will be clicking because 15 is a
How do you know?	multiple of 3
What will you be doing on the 20th beat?	On the 20th beat, I will be clapping
Explain your answer.	because 20 is not a multiple of 3



# Notes and Guidance

- Building on their knowledge of the two times table, children multiply by 4
- They link multiplying by 4 to doubling then doubling again. Children connect multiplying by 4 to repeated addition and counting in 4s.
- To show the multiplication of 4, children may use number pieces, cubes, counters, bar models etc.

# Mathematical Talk

- How many equal groups do we have?
- How many are in each group?
- How many do we have altogether?
- Can you write a number sentence to show this?
- Can you represent the problem in a picture?
- Can you use concrete apparatus to solve the problem?
- How many lots of 4 do we have?
- How many groups of 4 do we have?

# Varied Fluency





# Reasoning and Problem Solving

Tommy has four bags with five sweets in each bag.	Annie has more sweets.	Here is a blue strip of paper.	The blue strip is 4 cm long.
Annie has six bags with four sweets in each bag.	She has four more sweets than	An orange strip is four times as long.	The orange strip is 16 cm long.
Who has more sweets?	Tommy.		The orange strip is
How many more sweets do they have?		The strips are joined end to end.	4 times as long as the blue strip, so
Draw a picture to show this problem.		20 cm	there are 5 equal parts in total, and the length of each
		How long is the blue strip?	part is:
		How long is the orange strip?	20 ÷ 5 = 4 cm long.
		Explain how you know.	To find the length of the orange part:
			$4 \times 4 = 16$ cm.



#### **Notes and Guidance**

Children explore dividing by 4 through sharing into four equal groups and grouping in fours.

They use concrete and pictorial representations and their knowledge of the inverse to check their answers.

Mathematical Talk

Can you put the buttons into groups of fours?

- Can you share the number into four groups?
- What is the difference between sharing and grouping?

# Varied Fluency

Circle the buttons in groups of 4.



Can you also split the buttons into 4 equal groups? How is this the same? How is it different?

There are some cars in a car park. Fach car has 4 wheels. In the car park there are 32 wheels altogether. How many cars are there?

÷ =

Complete the bar models and the calculations. 24

4

4

4

4

4







# Reasoning and Problem Solving

Which of the word problems can be	No, the calculation	Five c	hildren are playin	g a game.		Mo = 4 buckets.
There are 12 bags of sweets with 4 sweets in each bag. How many sweets are there altogether?	IS $12 \times 4 = 48$ sweets Yes, 12 is being grouped into 4s	They knock	score 4 points for down.	every bucket th	ney	Eva = 7 buckets. Tommy = 3 buckets.
A rollercoaster carriage holds 4 people. How many carriages are needed for 12 people?	Yes, 12 is being shared equally into		Mo Eva	4 4 16 28		Amir = 8 buckets. Dora = 2 buckets.
I have 12 crayons and share them equally between 4 people. How many crayons does each person receive?	4 groups. No, the calculation	How r	Tommy Amir Dora nany buckets did	12 32 8 they knock dow	] /n	They knocked down 24 buckets altogether.
I have 12 buns and I give 4 to my brother. How many do I have left? Explain your reasoning for each.	is 12 – 4 = 8 buns	each? How r altoge How r down	nany buckets did ther? nany more bucke than Mo?	they knock dow	vn k	Eva knocked 3 more buckets down than Mo.

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#### The 4 Times Table

#### Notes and Guidance

Children use knowledge of known multiplication tables (2, 3, 5 and 10 times tables) and understanding of key concepts of multiplication to develop knowledge of the 4 times table.

Children who have learnt  $3 \times 4 = 12$  can use understanding of commutativity to know that  $4 \times 3 = 12$ 

# Mathematical Talk

What do you notice about the pattern?

Can you use concrete or pictorial representations to help you?

What other facts can you link to this one?

What other times tables will help you with this times table?

## Varied Fluency

Use the pictorial representations to complete the calculations.

$1 \times 4 =$	:
2 × 4 =	=

 $3 \times 4 =$ 



Continue the pattern.

2 cars have eight wheels. How many wheels do four cars have?

2 × 4 = 8 4 × 4 = \_\_\_\_

Three cows have 12 legs. How many legs do six cows have?

3 × \_\_\_\_ = 12 6 × \_\_\_\_

- 6 × \_\_\_\_ = \_\_\_\_
- Colour in the multiples of 4 What pattern do you notice?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50



#### The 4 Times Table

#### **Reasoning and Problem Solving**

I have forgotten what $4 \times 4$ is. $4 \times 4$ $= 3 \times 4$	<ul> <li>4</li> <li>4&lt;</li></ul>
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#### Notes and Guidance

Building on their knowledge of the 4 times table, children start to multiply by 8, understanding that each multiple of 8 is double its equivalent multiple of 4

They link multiplying by eight to previous knowledge of equal groups and repeated addition. Children explore the concept of multiplying by 8 in different ways, when 8 is the multiplier (first number in the multiplication calculation) and where 8 is the multiplicand (second number).

#### Mathematical Talk

How many equal groups do we have?

- How many are in each group?
- How many do we have altogether?
- Can you write a number sentence to show this?
- Can you represent the problem in a picture?
- Can you use concrete apparatus to solve the problem?
- How many lots of 8 do we have?
- How many groups of 8 do we have?
- We have 8 groups, how many are in each group?

## Varied Fluency



How many legs altogether do four spiders have? There are \_\_\_\_ legs on each spider.

If there are \_\_\_\_\_ spiders, there will be \_\_\_\_\_ legs altogether.

Arrange 24 counters in an array as shown and complete the calculations.





Fill in the table to show that multiplying by 8 is the same as double, double and double again.

		_						
6	6	6	6	6	6	6	6	
6 × 2	=	6 × 2		6 × 2	=	6 × 2	=	
	×2	2 =	× 2 =					
	× 2 =							



# Reasoning and Problem Solving

 $8 \times 3 = \_$   $2 \times 4 \times 3 = \_$  $2 \times 2 \times 2 \times 3 = \_$ 

What do you notice? Why do you think this has happened?

Jack calculates  $8 \times 6$  by doing  $5 \times 6$  and  $3 \times 6$  and adding them.

\_\_\_\_+ \_\_\_\_ = \_\_\_\_

Ron calculates  $8 \times 6$  by doing  $4 \times 6 \times 2$ 

\_\_\_\_ × 2 = \_\_\_\_

Whose method do you prefer? Explain why. All of the answers are equal. 8 has been split (factorised) into 2 and 4 in the second question and 2, 2 and 2 in the third. Possible answers: I prefer Jack's method because I know my 5 and 3 times tables. I prefer Ron's method because I know my 4 times table and can double numbers.

Start each function machine with the same number.



What do you notice about each final answer?

Tommy knows the 4 times table table, but is still learning the 8 times table table.

Which colour row should he use? Why?

Each time the final number is 8 times greater than the starting number.

Tommy should use the yellow row because he can double each multiple of 4 to calculate a number multiplied by 8 e.g. 4 × 6 = 24 so 8 × 6 is double that (48).



## Notes and Guidance

Children explore dividing by 8 through sharing into eight equal groups and grouping in eights.

They use concrete and pictorial representations and their knowledge of inverse operations to check their answers.

Mathematical Talk

What concrete/pictorial representations might help you?

Can you group the numbers in eights?

- Can you share the number into eights groups?
- Can you use any prior knowledge to check your answer?

#### Varied Fluency

There are 32 children in a PE lesson. They are split into 8 equal teams for a relay race. How many children are in each team? Use counters or multi-link to represent each child.

There are \_\_\_\_\_ teams with \_\_\_\_\_ children in each team.

Crayons are sold in packs of 8.
 Year 3 need 48 crayons.
 How many packs should be ordered?



They should order \_\_\_\_\_ packs of crayons.

Complete:

80 ÷ 8 = \_\_\_\_

64 ÷ 8 = \_\_\_\_ 8 × \_\_\_\_ = 40

x = 8 = 24 d = 7



#### **Reasoning and Problem Solving**

$48 \div 2 = \_$ $48 \div 4 = \_$ $48 \div 4 = \_$ $48 \div 8 = \_$ What do you notice about the answers to these questions? Can you predict what $48 \div 16$ would be? Which numbers can be divided by 8 without a remainder? $64 \qquad 32 \qquad 800$ $18 \qquad 200 \qquad 42$	The answers (quotients) halve and the divisors double. 3 64, 32, 800, 200	Amir shares 24 sweets equally between 8 friends. How many do they get each? Which bar model would you use to represent this problem? Why? 24 24 24	Although both can represent $24 \div 8 = 3$ , the first bar model fits this word problem best, because 24 has been split into 8 parts, 1 part shows 1 friend.
	12	0	







#### The 8 Times Table

# **Reasoning and Problem Solving**

All the numbers in the 8 times table are even.

#### Explain why

On a blank hundred square, colour multiples of 8 red and multiples of 4 blue.

#### Always, Sometimes, Never

- Multiples of 4 are also multiples of 8
- Multiples of 8 are also multiples of 4

When you add an even number to an even number you always make an even number. The 8 times table is repeated addition so keeps adding an even number each time. 1) Sometimes, every other multiple of 4 is also a multiple of 8 The ones in between aren't because the jumps are smaller than 8 2) Always - 8 is a multiple of 4 therefore all multiples of 8 will be multiples of 4

Rosie has some packs of cola which are in a box.

Some packs have 4 cans in them, and some packs have 8 cans in them.

0

C O



Rosie's box contains 64 cans of pop.

How many packs of 4 cans and how many packs of 8 cans could there be?

Find all the possibilities.

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Possible answers:

- 2 packs of 4, 7 packs of 8
- 4 packs of 4, 6 packs of 8
- 6 packs of 4, 5 packs of 8
- 8 packs of 4, 4 packs of 8
- 10 packs of 4, 3 packs of 8
- 12 packs of 4, 2 packs of 8
- 14 packs of 4, 1 pack of 8